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Math 406, Spring 2021
HW07, due Wednesday, March 31 at 5pm [1]

Reading: Read Chapters 8 and 9 of Crisman's text.

Graded Problems: Work the following problems for a grade. Turn them in on Gradescope.

Some problems are taken from the Online Version of Crisman's text:

`http://math.gordon.edu/ntic/`

Each problem is worth 20 points.

1 (Crisman 9.6.8)**.** Evaluate $\mathrm{Mod}(11^{49},21)$ and $\mathrm{Mod}(139^{112},27)$ without using a computer or a calculator. (And show your work.)

Here, when n is a positive integer, $\mathrm{Mod}(k,n)$ is the unique integer r such that $0 \le r < n$ and $k = qn + r$ for some integer q. In other words, $\mathrm{Mod}(k,n)$ is the remainder you get when you divide n into k.

2. Suppose n is an integer with $n > 1$, and let S denote the set of all prime numbers appearing in the prime factorization of n. Show that

$$\phi(n) = n \prod_{p \in S} 1 - 1/p.$$

3. As in class, when d is a positive integer write

$$S_d := \{k \in \mathbb{Z} : 0 < k \le d, \gcd(k,d) = 1\}.$$

Then $\phi(d)$ is, by definition, the cardinality $\#S_d$ of S_d.

Suppose n and m are positive integers which are coprime, and suppose that $a \in S_n$, $b \in S_m$. Show that there exists a unique integer $x \in S_{nm}$ such that

$$\mathrm{Mod}(x,n) = a$$
$$\mathrm{Mod}(x,m) = b$$

This will finish the proof I gave in class for the multiplicativity of Euler's function ϕ.

4 (Crisman 9.6.13)**.** Compute the ϕ function evaluated at 1492, 1776, and 2001.

5 (Crisman 8.4.1-2)**.** Write out the addition and multiplication tables for the ring $\mathbb{Z}_{11}$ by hand.

[1] This version created Tuesday 27$^{\text{th}}$ April, 2021 at 21:42.