## Hint for Problem 3:

1. If $u(t)$ is differentiable between $t_1, t_2$ we have

$$u(t_2) - u(t_1) = \int_{t_1}^{t_2} u'(s)ds$$
$$|u(t_2) - u(t_1)| \le |t_2 - t_1| \max_{s\in[t_1,t_2]} |u'(s)|$$

2. If $u(t)$ is analytic at $t_0$ we have near $t_0$ the Taylor expansion

$$u(t) - u(t_0) = u'(t_0)(t - t_0) + \frac{u''(t_0)}{2!}(t - t_0)^2 + \cdots$$

   If $u'(t_0) \neq 0$: E.g., for $u'(t_0) > 0$ we have for $t \ge t_0$

$$(u(t) - u(t_0))^{1/3} = (t - t_0)^{1/3} \underbrace{\left[u'(t_0) + \frac{u''(t_0)}{2!}(t - t_0) + \cdots\right]}_{=:\, q(t)}^{1/3}$$

   where the function $q(t)$ is analytic with $q(t_0) > 0$.

3. For $s < t_0 < t$

$$|u(s) - u(t)| \le |u(s) - u(t_0)| + |u(t_0) - u(t)| \le C\,|s - t_0|^\alpha + C\,|t - t_0|^\alpha \le C2^{1-\alpha}\,|s - t|^\alpha$$

   since $x^\alpha$ is concave: $\left(\frac{a}{2} + \frac{b}{2}\right)^\alpha \ge \frac{1}{2}\left(a^\alpha + b^\alpha\right)$ for $0 < \alpha \le 1$, $a, b \ge 0$.