

In this assignment we investigate functions of two variables. The purpose of the problems is to produce some nice pictures. Don't forget to label your graphs.

1. We wish to graph  $f(x, y) = x^2 - y^2$  over the square  $\{-2 \leq x, y \leq 2\}$ . We do this as follows:

```
f=@(x,y) x.^2-y.^2;
% Set up a mesh for plotting.
x=-2:.1:2; y=-2:.1:2;
[X,Y]=meshgrid(x,y);
Z=f(X,Y);
% The plotting command is surf.
surf(X,Y,Z)
```

2. Repeat problem 1 for

- (a)  $f(x, y) = \sin(x + y)$ .
- (b)  $f(x, y) = \cos(x^2 + y^2)$ .

3. Repeat problem 1 using the m-file **qsurf**. To see how to use it type `help qsurf`. Use a few different values of  $n$ .

4. We can also plot contour lines using the command **contour**. Let  $X, Y, Z$  be as in problem 1. Now do

```
level=-1.5:3:1.5;
contour(X,Y,Z,level)
```

5. We find the tangent plane approximation to  $f(x, y) = (1 - y^2)(1 - x^2)$  at the point  $(x_0, y_0) = (.2, -.4)$ . The partial derivatives are  $f_x(x, y) = -2x(1 - y^2)$  and  $f_y(x, y) = -2y(1 - x^2)$ . Hence the tangent plane to the graph of  $f$  at  $P_0 = (.2, -.4, f(.2, -.4))$  is

$$\begin{aligned} z = l(x, y) &= f(.2, -.4) + f_x(.2, -.4)(x - .2) + f_y(.2, -.4)(y + .4) \\ &= .8064 - .336(x - .2) + .768 * (y + .4), \end{aligned}$$

which has the normal vector

$$\mathbf{N} = [-f_x(x_0, y_0), -f_y(x_0, y_0), 1] = [.336, -.768, 1].$$

Now we graph  $f$  over the square  $\{-1 \leq x, y \leq 1\}$  and attach the tangent plane and normal vector. We graph the tangent plane over the smaller square  $\{|x - .2|, |y + .4| \leq .5\}$ , and use a coarser mesh to make it more visible.

```
f= @(x,y) (1-x.^2).*(1-y.^2);
l=@(x,y) .8064-.336*(x-.2)+.768*(y+.4);
qsurf(f, [-1, 1, -1, 1])
hold on
```

```

qsurf(1, [-.3, .7, -.9, .1], 10)
P=[.2, -.4, f(.2, -.4)]; N = [.336, -.768, 1]; arrow3(P,N,'r')
hold off

```

6. We will now display a contour plot along with the gradient vector field. To display a vector field we use the command **quiver**.

Let  $f(x, y) = xy - x^3/3$ . Then  $f_x(x, y) = y - x^2$  and  $f_y = x$ . We shall display the gradient vector field and the level curves of  $f$  over the square  $[-2, 2] \times [-2, 2]$ .

```

f=@(x,y) x.*y-(x.^3)/3;
fx=@(x,y) y-x.^2;
fy=@(x,y) x;
x=-2:.05:2;y=x;
% this is the fine mesh for the level curves.
[X,Y]=meshgrid(x,y);
Z=f(X,Y);
% We choose the level curves.
levels = [-6:.5:6];
contour(X,Y,Z,levels)
hold on
xx=-2:.2:2; yy=xx;
% This is the coarse mesh for the arrows
[XX,YY]=meshgrid(xx,yy);
U=fx(XX,YY); V=fy(XX,YY);
quiver(XX,YY,U,V)
axis equal

```

What is the relation between the level curves and the arrows?

7. Repeat problem 6 for  $f(x, y) = x^2 + 4y^2$  Use the same square but you will need to consider a different set of level curves.