AMSC/CMSC 460 Dr. Wolfe ASSIGNMENT #3 Due June 26, 2009

1. The Runge function is

$$r(x) = \frac{1}{1+25x^2}, \quad -1 \le x \le 1.$$

   (a) For $n = 5, 10, 15$, plot $p_n(x)$, the polynomial interpolating $r(x)$ at $n+1$ equally spaced points, along with the graph of $r(x)$. Use the MATLAB functions POLYFIT and POLYVAL. Observe what is happening to the graphs. Where is the polynomial fit getting better ? Where is it getting worse ?

   (b) Repeat part (a) but now use the interpolation points

$$x_j = \cos\frac{(2j-1)\pi}{2n+2}, \quad j = 1, \ldots, n+1.$$

   What difference do you observe ?

2. Given the data $(-1, 2), (0, 2), (1, 2), (2, 5)$, calculate $P_3(x)$, the cubic polynomial interpolating this data (by hand) in three ways:
   (a) Solve the Vandemonde system.
   (b) Use Lagrange Polynomials.
   (c) Find the Newton form, using the divided difference table.
   Show that you get the same polynomial in each case.

3. For $f(x) = \sinh x$ we are given that

$$f(0) = 0,\ f'(0) = 1,\ f(1) = 1.1752,\ f'(1) = 1.5431.$$

   Calculate an approximation to $f(0.5)$ using cubic Hermite interpolation. Compare the result with $f(0.5) = .5211$.

4. Consider the function S(x) defined as

$$S(x) = \begin{cases} 28 + 25x + 9x^2 + x^3, & -3 \le x \le -1, \\ 26 + 19x + 3x^2 - x^3, & -1 \le x \le 0, \\ 26 + 19x + 3x^2 - 2x^3, & 0 \le x \le 3, \\ -163 + 208x - 60x^2 + 5x^3, & 3 \le x \le 4. \end{cases}$$

   Show that $S(x)$ is a natural cubic spline function with the knots $\{-3, -1, 0, 3, 4\}$. (A natural cubic spline is a spline $S(x)$ which satisfies $S''(x_1) = S''(x_N) = 0$) Be sure to state explicitly each of the properties of $S(x)$ which are necessary for this to be true.

5. The vapor pressure P of water (in bars) as a function of temperature T (°C) is

T	0	10	20	30
P(T)	.006107	.012277	.023378	.042433
T	40	50	60	70
P(T)	.073774	.12338	.19924	.31166
T	80	90	100	110
P(T)	.47364	.70112	1.01325	1.22341

Interpolate these data with the cubic spline $S(x)$ using the MATLAB function SPLINE and plot the results. It is also known that $P(5) = .008721, P(45) = 0.095848$ and $P(95) = 0.84528$. How well does $S(x)$ do at these points ?

6. Ex.3.3 p.110, *Numerical Computing with MATLAB* .
7. Ex.3.16 p.116, *Numerical Computing with MATLAB* .
8. Ex.3.17 p.116, *Numerical Computing with MATLAB* .
9. Ex.5.8 p.161, *Numerical Computing with MATLAB* .
10. Ex 5.12 p.164, *Numerical Computing with MATLAB*
11. For the CENSUSGUI data on p.144 find the cubic polynomial $p_3(s) = \beta_1 s^3 + \beta_2 s^2 + \beta_3 s + \beta_4$, where s is the translated and scaled time variable, which interpolates the data in the sense of least squares by constucting the 11×4 data matrix A and finding the vector $(\beta_1, \beta_2, \beta_3, \beta_4)^T$ in four different ways:
    (a) By using the backslash operator.
    (b) By forming and solving the normal equations. Note the condition number of the matrix $A^T A$.
    (c) By using the QR decomposition.
    (d) By using the Singular-Value Decomposition.
    All of this is quite easy in MATLAB. Compare with the values given by POLYFIT.