MATH 463: HOMEWORK ASSIGNMENT # 1: SOLUTIONS

2.1

(a) $(\sqrt{2}-i)-i(1-\sqrt{2}i)=\sqrt{2}-i-i+\sqrt{2}i^2=\sqrt{2}-2i-\sqrt{2}=-2i$

(d) Multiplying the numerator and denominator of the first fraction by $3+4i$ and the second by $-5i$, both denominators become 25 and we obtain

$$\frac{1}{25}((1+2i)(3+4i)-10i+5))=\frac{1}{25}(-5+10i-10i-5)=-\frac{2}{5}.$$

(e) Multiplying numerator and denominator by the complex conjugate of the denominator, we obtain

$$\frac{5(1+i)(2+i)(3+i)}{100}=\frac{50i}{100}=\frac{i}{2}.$$

(f) $(1-i)^4=1-4i+6i^2-4i^3+i^4=1-4i-6+4i+1=-4.$

4.4 Setting $z=x+yi$, we have $\Re(z)=x$, $\Im(z)=y$, and

$$2|z|^2=2(x^2+y^2)=(|x|+|y|)^2+(|x|-|y|)^2\geq(|x|+|y|)^2.$$

Since all terms are positive, taking square roots gives the desired result.

4.10 Your answer to (a) should be a circle of radius 1 centered at $1-i$. Your answer to (b) should be a disk (a circle with the interior filled in) of radius 3 centered at $-i$. Your answer to (c) should be the line $x=2$, since z, $\bar{z}$, and $\bar{z}-i$ all have the same real part. Your answer to (d) should be a circle of radius 2 centered at $\frac{i}{2}$.

4.12 $z^4-4z^2+3=(z^2-1)(z^2-3)$. If $|z|=2$, $|z^2|=4$, and it follows from inequality (5) that $|z^2-1|\geq|4-1|=3$ and $|z^2-3|\geq|4-3|=1$. It now follows that

$$\left|\frac{1}{z^4-4z^2+3}\right|\leq\frac{1}{|z^2-1|}\frac{1}{|z^2-3|}=\frac{1}{3}.$$

6.12 There are two things to prove. The first is that $|c_1c_2|$ and $c_1\bar{c_2}$ have the same modulus. This follows immediately from the fact that c_2 and $\bar{c_2}$ have the same modulus and the fact that the modulus of a product is the product of the moduli.

The second thing to prove is that if $|z_1| = |z_2|$, then c_1 and c_2 exist with $z_1 = c_1c_2$ and $z_2 = c_1\bar{c_2}$. This is easier than it looks at first. Set $r = |z_1| = |z_1|$. Let θ_1 and θ_2 be determined by the equations $\theta_1 + \theta_2 = \text{Arg}(z_1)$, $\theta_1 - \theta_2 = \text{Arg}(z_2)$. Then set $c_1 = re^{i\theta_1}$ and $c_2 = e^{i\theta_2}$. This is not the only solution; the moduli of c_1 and c_2 can be any two positive numbers whose product is r. I chose r and 1; you might have chosen $\sqrt{r}$ for both moduli, and of course there are many other choices.

supp1.1

(a) $(3 + 4i)(2 + 7i) = -22 + 29i$

(b) $\frac{2+i}{3+i} = \frac{(2+i)(3-i)}{10} = \frac{7+i}{10}$

supp1.2 If we set $x + yi$ to be a square root of $3 + 4i$, we obtain

$$(x + yi)^2 = x^2 - y^2 + 2xyi = 3 + 4i.$$

This gives $2xy = 4$ and $x^2 - y^2 = 3$. From the first of these equations, we get $y = \frac{2}{x}$. Substituting this in the second gives $x^2 - \frac{4}{x^2} = 3$. This simplifies to $x^4 - 3x^2 - 4 = (x^2 - 4)(x^2 + 1) = 0$. Since we know x is real, x^2 must be positive. This gives $x^2 = 4$, giving $x = \pm 2$; $y = \frac{2}{x}$ it follows that the square roots we want are $\pm(2 + i)$.

supp1.3 To solve the various parts of this problem it will be useful to begin by observing that the principal argument of $1 + i$ is $\frac{\pi}{4}$ while the principal argument of $2 + i$ is $\arctan \frac{1}{2}$, which is roughly .4636 or, more conveniently, $.1476\pi$. This solves part (a) and tells us that an argument of $\frac{1+i}{2+i}$ is $\text{Arg}(1+i) - \text{Arg}(2+i) \approx .1124\pi$, while an argument of $(1+i)^3$ is $\frac{3\pi}{4}$. Note that all of these are in the principal argument range and so give the desired answers to parts (b) and (c).