MATH 463: HOMEWORK ASSIGNMENT # 3: SOLUTIONS

14.4 We can rewrite $(\frac{z}{\bar{z}})^2$ as $e^{4i\mathrm{Arg}(z)}$. From this, it is evident that

$$\lim_{r\to 0} f(re^{i\theta})$$

exists for every θ, but is not independent of θ. It follows that the limit of $f(z)$ as z approaches 0 does not exist.

16.1 These are routine exercises in differentiation, using the usual differentiation rules.

(a) $f'(z) = 6z - 2$
(b) $f'(z) = 8z(1 - 4z^2)^2$
(c) $f'(z) = \frac{3}{(2z+1)^2}$
(d) $f'(z) = \frac{(1+z^2)^3(4z^3-2z)}{z^4}$

16.9 This is closely related to problem 14.4. At $z = 0$ the difference quotient $\dfrac{\Delta w}{\Delta z}$ can be written as $e^{-4i\mathrm{Arg}(\Delta z)}$, so that the limit fails to exist for the same reason as in problem 14.4.

supp 7. The limiting value of $f(z)$ at both a1 and -1 is ∞, since the denominator approaches 0 and the numerator does not. The limiting value at ∞ is 1, as may be seen easily by dividing both numerator and denominator by z^2.

supp 8. If p has higher degree than q, the limiting value is ∞. If p has lower degree than q, the limiting value at ∞ is 0. If p and q have equal degree, the limiting value at ∞ is the ratio of the leading coefficient of p to the leading coefficient of q.