MATH 463: HOMEWORK ASSIGNMENT # 8: SOLUTIONS

43.2 $r_n = \sqrt{4 + \frac{1}{n^4}}$, which clearly converges to 2. Since z_n is alternately in the second and third quadrants, Θ_{2n} converges to π and Θ_{2n+1} converges to $-\pi$. It follows that Θ_n does not converge.

45.6 The derivatives of $\cos(z)$ at $\frac{\pi}{2}$ (starting with the value or zeroth derivative) are $0, -1, 0, 1, 0, -1, \cdots$ This gives the power series

$$\sum_{0}^{\infty} \frac{(-1)^{n+1}}{(2n+1)!}(z - \frac{\pi}{2})^{2n+1}$$

45.8 $\tanh(z) = \dfrac{\sinh(z)}{\cosh(z)}$. Both numerator and denominator are entire, so the radius of convergence is distance from 0 to the nearest zero of the denominator, which is $\dfrac{\pi}{2}i$. Consequently, the radius of convergence is $\dfrac{\pi}{2}$. To calculate terms of the series, we perform the long division keeping only sufficiently many terms of each series to determine the first two terms of the quotient.

$$\frac{z + \frac{z^3}{6}}{1 + \frac{z^2}{2}} = z - \frac{z^3}{3} + \cdots$$

supp14

(a) For $|z| < 1$, we have the geometric series

$$\sum_{n=0}^{\infty} (-1)^n z^{2n}$$

(b) For $0 < |z - i| < 2$, we set

$$\frac{1}{1+z^2} = \frac{1}{z-i}\frac{1}{(z-i)+2i} = \frac{1}{2i(z-i)}\frac{1}{1+\frac{z-i}{2i}}$$

We can now expand the last term as a geometric series to obtain

$$\sum_{n=0}^{\infty} (-1)^n \frac{(z-i)^{n-1}}{(2i)^{n+1}}$$

(c) We proceed in much the same manner, to obtain

$$\frac{1}{1+z^2} = \frac{1}{z+i}\frac{1}{(z+i)-2i} = -\frac{1}{2i(z+i)}\frac{1}{1-\frac{z+i}{2i}}$$

which yields the series

$$-\sum_{n=0}^{\infty}\frac{(z+i)^{n-1}}{(2i)^{n+1}}$$

(d) In this case, we set

$$\frac{1}{1+z^2} = \frac{1}{z^2}\frac{1}{1+\frac{1}{z^2}} = \sum_{n=0}^{\infty}(-1)^n z^{-(2n+2)}$$