MATH 463 (PROFESSOR GREEN) SUPPLEMENTARY PROBLEM SET #1

1. Express each of the following in the form $a + bi$ with a and b real.
 (a) $(3 + 4i)(2 + 7i)$
 (b) $\dfrac{2+i}{3+i}$

2. Express $\sqrt{3 + 4i}$ in the form $\pm(x+yi)$ with x and y real by solving a system of quadratic equations for x and y as in Problem 12 of Section 2 in your text.

3. Find the principal argument Arg$z$ when
 (a) $z = 2 + i$
 (b) $z = \dfrac{1+i}{2+i}$
 (c) $(1 + i)^3$

4. Adapt the ideas in Problem 4 of Section 7 to express the square roots of $a + bi$ in the form $\pm(x + yi)$, with a, b , x and y all real. You will need to consider separately the cases where $b = 0$, $b > 0$, and $b < 0$.

5. Write the function $f(z) = z^3 + z^2 + 1$ in the form $f(z) = u(x, y) + iv(x, y)$ where $z = x + yi$ and u and v are real functions of x and y.

6. Express $xy + i(x^2 + y^2)$ as a function of $z = x + yi$ and $\bar{z} = x - yi$.

7. Let $f(z) = \dfrac{z^2+1}{z^2-1}$. Calculate the limiting values of $f(z)$ at $x = \pm 1$ and $z = \infty$.

8. Let $p(z)$ and $q(z)$ be polynomials. State a rule for calculating the limiting value of $\dfrac{p(z)}{q(z)}$ at $z = \infty$.

9. For each of the following functions of $z = x + iy$, find its singular points, find its real and imaginary parts, $u(x, y)$ and $v(x, y)$, and verify directly that u and v are harmonic.

(a) $f(z) = \dfrac{z}{z^2 + 1}$

(b) $f(z) = \dfrac{z^2 + 2}{z^2 - 4}$

10. Determine which of the following functions are entire; explain in each case how you know.

(a) $f(x + iy) = e^x \cos y + ie^x \sin y$.

(b) $f(x + iy) = x^2 + 2iy^2$

(c) $f(z) = \dfrac{z + 1}{z + 2}$

11. Find all roots of each of the following equations.

(a) $\sin z = \frac{1}{2}$

(b) $\cosh z = i$

(c) $\log z = 2 + \dfrac{\pi}{3} i$