STAT 100 SUMMER II 2001 (PROFESSOR GREEN) SOLUTIONS TO ASSIGNED PROBLEMS DUE AUGUST 17

Problem 12

(a) $\bar{x} = 1.02; \quad s = .26$

(b) The sample size is 9, so there are eight degrees of freedom. $t_{.01} = 2.896$, so the error bound is $d = \frac{2.896 \times .26}{\sqrt{8}} = .27$. This gives a confidence interval of $(.75, 1.29)$.

Problem 16.

(a) The point estimate is the midpoint of the confidence interval which is the average of the two endpoints, in this case 134.

(b) The 90% error bound is 12. To obtain the95% error bound, one must replace $t_{.05} = 1.740$ (There are 17 degrees of freedom) with $t_{.025} = 2.110$ in the formula for the error bound. This boils down to dividing the 90% error bound by 1.740 and multiplying by 2.110, giving $\frac{12}{1.740} \times 2.110 = 14.55$, giving a 95% confidence interval of $(119.45, 148.55)$.

Problem 22. The data give a mean of 26.6 and a standard deviation of 6.56. There are 13 measurements, so there are 12 degrees of freedom. The null hypothesis is $\mu = 21$; the alternative hypothesis is $\mu > 21$. The test statistic is given by

$$T = \frac{\bar{X} - 21}{\frac{S}{\sqrt{13}}}.$$

The rejection region is $T \geq t_{.01} = 2.681$. The value of the test statistic for the sample to hand is given by $t = \frac{(26.6-21) \times \sqrt{13}}{6.56} = 3.08$. Since 3.08 is in the rejection region, the null hypothesis is rejected in favor of alternative hypothesis that the mean diameter of the mounds is greater than 21 feet.

Problem 28. Writing μ for the current average size of a farm in acres, the null hypothesis is $\mu = 160$, and the alternative hypothesis is $\mu > 160$. The test statistic, with a sample size 27 is given by

$$T = \frac{\mu - 160}{\frac{S}{\sqrt{27}}}.$$

The value of the test statistic from the data is $t = 2.8868$ which, with 26 degrees of freedom, gives a P-value between .001 and .005. Since this is a very low P-value, the sample provides strong evidence that the average size of farms has increased. For the confidence interval, we have $t_{.01} = 2.473$ so that $d = \frac{(36 \times 2.473)}{\sqrt{27}} = 17.13$ This gives a 98% confidence interval of $(162.87, 197.13)$ for the current average size.

Problem 30. The cadmium concentration had mean 18 and standard deviation 10.67. The zinc concentration has mean 151.67 and standard deviation 21.37. Both sample sizes are 6, so the estimated standard errors are 4.36 for the cadmium and 8.72 for the zinc.

(a) With 5 degrees of freedom $t_{.025} = 2.571$ giving error bounds of 11.21 for the cadmium and 22.43 for the zinc. This gives confidence intervals of $(6.79, 29.21)$ for the cadmium and $(129.24, 174.10)$ for the cadmium. Since both variables are inherently positive, the relatively large standard deviation for the cadmium casts serious doubt on the normality of the distribution and, therefore, on the validity of this computation. This also applies to Part (b).

(b) Here, the null hypothesis is that the mean concentration of cadmium is 12; the alternative hypothesis is that it is higher. The value of the test statistic in this case is $t = \frac{18-12}{6.16} = 1.38$. With 5 degrees of freedom this gives a P-value larger that .1 which does not provide very strong support for the hypothesis that the mean concentration of cadmium is higher than 12. However, in any case, this entire test is suspect because the data call the normality of the cadmium concentration seriously in question.