STAT 100 SUMMER II 2001 (PROFESSOR GREEN) SOLUTIONS TO ASSIGNED PROBLEMS DUE AUGUST 20

Problem 6. The point estimate for the difference of the means is $81.2 - 76.4 = 4.8$. The estimated standard error is $\sqrt{\frac{8.2^2}{90} + \frac{7.6^2}{100}} = 1.09$ Since $z_{.01} = 2.33$, the error bound is $1.09 \times 2.33 = 2.55$. This gives $(2.25, 7.35)$ as the desired confidence interval.

Problem 8. This time the point estimate is $108 - 89 = 19$ and the estimated standard error is $\sqrt{\frac{46.2^2}{78} + \frac{53.4^2}{62}} = 8.56$. The error bound is $8.56 \times 2.33 = 19.96$, giving a confidence interval of $(-0.96, 38.96)$. The meaning of the minus sign is that we cannot say with 98% confidence that the mean for Treatment 2 is higher than the mean for Treatment 1.

Problem 16.

(a) $s_{pooled} = \sqrt{\frac{28+22}{13+14-2}} = 1.414$.

(b) The estimated standard error is $E.S.E. = s_{pooled} \times \sqrt{\frac{1}{14} + \frac{1}{13}} = .545$. Because the sample sizes are relatively small, we will use a t variable with 25 degrees of freedom. The test statistic is $T = \frac{\bar{X}-\bar{Y}}{E.S.E}$ and the rejection region is $T > t_{.05} = 1.708$. The actual value of the test statistic is $t = \frac{20-17}{.545} = 5.51$, which is well into the rejection region and, in fact, gives a P-value that is much less than .0005. The null hypothesis is rejected in favor of the alternative hypothesis.

(c) The point estimate is $20 - 17 = 3$ the error bound is $d = E.S.E. \times t_{.025} = .545 \times 2.06 = 1.12$. The desired confidence interval is $(1.88, 4.12)$.

Problem 18.

(a) Here, the sample sizes are just large enough to use a Z-test. The t-test with pooling is also possible but would give only very slightly more conservative results.

(b) The sample sizes are small, but the standard deviations are reasonably close. The t-test with pooling is the best choice.

(c) The sample sizes are small, so a t-test is required. The standard deviations are too far apart for pooling, so the conservative t-test without pooling is the only appropriate one.
(d) The sample sizes are well into the range that justifies the Z-test. If we were to use a t-test, the standard deviations are much to far apart to justify pooling, but even the conservative 59 degrees of freedom would not give a significantly different result from the Z-test.

Problem 30. The standard deviations are close enough to justify pooling. $s_{pooled} = \sqrt{\frac{8\times 8.16^2 + 8\times 6.93^2}{16}} = 7.57$. The estimated standard error is given by $E.S.E. = 7.57\sqrt{\frac{2}{9}} = 3.57$.

(a) The test statistic is $T = \frac{\mu_c - \mu_s}{E.S.E}$ and the rejection region is $T > t_{.05} = 1.746$ with 16 degrees of freedom. The actual value of the test statistic is $\frac{39.8-35.5}{3.57} = 1.20$. This is not in the rejection region, and the null hypothesis is retained. Note that this does not mean that the null hypothesis is affirmed; the test is inconclusive, and makes no statement about which activity rate is higher.
(b) The error bound is given by $d = E.S.E. \times t_{.025} = 3.57 \times 2.12 = 7.57$ The point estimate for $\mu_c - \mu_s$ is $39.8 - 35.5 = 4.3$ and the confidence interval is therefore $(-3.27, 11.87)$. The opposite signs of the lower and upper limit reflect the fact that, at the given confidence level, one cannot make a statement as to which activity level is higher.