STAT 100 SUMMER II 2001 (PROFESSOR GREEN) SOLUTIONS TO ASSIGNED PROBLEMS DUE AUGUST 21

Problem 36. There are three degrees of freedom . Taken in the sense of $Y - X$, $\bar{d} = 1.25$, the standard error is 1.28, and $t = .976$.

Problem 38.

(a) From the data, we have $\bar{d} = 6.25, \quad S_D = 9.8$, so, with $n = 12$, the estimated standard error is 2.83. This gives us a t-statistic of 2.209 with 11 degrees of freedom. This gives a P-value slightly smaller than .025, so at the significance level .05, the null hypothesis that there is no difference in the mean responses to the treatments is rejected in favor of the alternative hypothesis that the mean response of treatment A is greater.

(b) The point estimate is 6.25 and the error bound is $E.S.E \times t_{.025} = 2.83 \times 2.201 = 6.23$. This gives a confidence interval of $(.02, 12.48)$.

Problem 42.

(a) The mean difference, taken in the sense $B - A$ is -0.3 and the standard deviation is .45. With $n = 6$, this gives a standard error of .184 and, with 5 degrees of freedom, a 95% error bound of .47. This gives a confidence interval for the mean difference of $(-.77, .17)$.

(b) Instead of giving medicine A the first night and B the second, one might randomize which medicine is given on which night. This would tend to average out any lingering effect of the medicine given on the first night.

Problem 44.

(a) The sample mean decrease is 42.9. The sample standard deviation of the decrease is 34.35 which, with a sample size of 10 gives an estimated standard error of 10.86. With nine degrees of freedom $t_{.025} = 2.262$, giving an error bound of 24.57, and thus a confidence interval of $(14.33, 65.47)$.

(b) We have seen above that 0 is not in the 95% confidence interval for δ. This means that the null hypothesis $\delta = 0$ will be rejected in favor of the alternative $\delta \neq 0$ at that confidence level.

(c) In order to randomize, the experiment would have to be decided on a year in advance. Then the participants could be randomized to be offered the lower rates either in the first year of the experiment or in the second year.

(d) Without randomization, one has to worry about the way in which the difference may be affected by year to year differences in climate, the price of electricity etc. An alternative to randomization is to set up a control group that is not offered the lower off peak rates, and use independent sample techniques to compare the year-to year mean difference in the two groups.

Problem 50. The observed proportions are $\hat{p}_A = .62$ and $\hat{p}_B = .31$.

(a) The pooled estimate for $p_A = p_B$ assuming the null hypothesis that $p_A = p_B$ is given by $p = \frac{93}{200} = .465$. On that assumption, the test variable is given by

$$Z = \frac{p_A - p_B}{\sqrt{.465 \times .535 \times \frac{2}{100}}}.$$

The actual test statistic is then 4.4, which has an extremely low P-value and certainly rejects the null hypothesis at significance level .05.

(b) The observed difference is .31 The estimated standard error, based on the observed proportions is $\sqrt{\frac{.62\times.38}{100} + \frac{.31\times.69}{100}} = .067$. This gives a 95% error bound of $.067 \times 1.96 = .1314$ and a confidence interval for the difference of $(.1786, .4414)$.

Problem 60. The observed difference is $\frac{173}{250} - \frac{120}{250} = .212$. The estimated standard error, based on the data is .043. This gives a 95% error bound of $.043 \times 1.96 = .084$. The desired confidence interval is $(.128, .296)$.