STAT 100 SUMMER II 2001 (PROFESSOR GREEN) SOLUTIONS TO ASSIGNED PROBLEMS DUE AUGUST 7

Problem 40.

(a) From the binomial table,

$P[X = 9] = .425 - .274 = .151$

$P[7 \leq X \leq 13] = .922 - .074 = .848$

$P[7 < X < 13] = .846 - .154 = .692$

(b) X has mean 10, variance 6, and standard deviation $\sqrt{6} = 2.45$. As in class, we write $\hat{X}$ for the approximating normal variable.

$P[X = 9] \approx P[8.5 < \hat{X} < 9.5] = P[-.61 < Z < -.20] = .4207 - .2709 = .1498$

$P[7 \leq X \leq 13] \approx P[6.5 < \hat{X} < 13.5] = P[-1.43 < Z < 1.43] = .9236 - .0764 = .8472$

$P[7 < X < 13] \approx P[7.5 < \hat{X} < 12.5] = P[-1.02 < Z < 1.02] = .8461 - .1539 = .6922$

Notice that in each case the normal approximation, rounded off to three decimal places, is within ,001 of the value from the table.

Problem 42. X and the approximating normal distribution $\hat{X}$ both have mean 130, variance 45.5, and standard deviation $\sqrt{45.5} = 6.75$.

(a) $P[X = 120] \approx P[119.5 < \hat{X} < 120.5] = P[-1.56 < Z < -1.41] = .0793 - .0594 = .0199$

(b) $P[X \leq 140] \approx P[\hat{X} < 140.5] = P[Z < 1.56] = .9406$

(c) $P[117 \leq X \leq 142] \approx P[116.5 < \hat{X} < 142.5] = P[-2.00 < Z < 1.85] = .9678 - .0228 = .9450$

Problem 46. Let X be the number of residents in the survey under the age of 34.9 years. Then X is a binomial variable of type $(200, .5)$, and has mean 100, variance 50 and standard deviation $\sqrt{50} = 7.07$. We want $P[X \geq 110] \approx P[\hat{X} > 109.5] = P[Z > 1.34] = .0901$

Problem 50. Let X be the number of weeks during the next year in which the company's travel expenses will exceed \$870. X is a binomial variable of type $(52, p)$ where $p = P[Y > 870]$, and Y represents the travel expenses in a randomly chosen week. We need to compute

$P[X \geq 20]$, but we must first compute p. Y has mean 850 and standard deviation 40 so that

$$p = P[Y > 870] = P[Z > .5] = .3085.$$

Now we can say that X had mean $52p = 16.04$, and standard deviation $\sqrt{52p(1-p)} = 3.33$. Now we can estimate

$$P[X \geq 20] \approx P[\hat{X} > 19.5] = P[Z > 1.04] = .1492.$$

Problem 52. This problem calls for $P[70 \leq X \leq 95]$ where X is a binomial variable of type $(200, .4)$. X has mean 80, variance 48, and standard deviation 6.93. Accordingly,

$$P[70 \leq X \leq 95] \approx P[69.5 < \hat{X} < 95.5] = P[-1.52 < Z < 2.24] = .9875 - .0643 = .9232.$$

Problem 54. X is the number of trees in the sample that have the parasite. X is binomial of type $(300, .2)$, and has mean 60, variance 48, and standard deviation 6.93.

(a) $P[49 \leq X \leq 71] \approx P[48.5 < \hat{X} < 71.5] = P[-1.66 < Z < 1.66] = .9515 - .0485 = .9030$.

(b) $P[X \geq 72] \approx P[\hat{X} \geq 71.5] = P[Z > 1.66] = .0485$. The computation shows that, If the proportion of infested trees were really .2, a sample with this high a level of infestation would be rather improbable. For this reason, such a result would provide reasonably strong support for the claim that the real proportion is higher than .2