STAT 100 SUMMER II 2001 (PROFESSOR GREEN) SOLUTIONS TO ASSIGNED PROBLEMS DUE AUGUST 8

Problem 10. The expected value of the sample mean for all sample sizes is 25. The standard deviation for the sample mean is $\frac{14}{3}$ for samples of size nine and $\frac{14}{4}$ for samples of size sixteen.

Problem 16. $\bar{X}$ has mean 30 and standard deviation $\frac{7}{\sqrt{6}} = 2.86$. Because the parent distribution is normal, $\bar{X}$ is also normally distributed, even though the sample size is small.

Problem 18. We are told here to assume that the parent distribution is normal with mean 16.08 and standard deviation .122. Even though we are dealing with a small sample size, we may assume the sample mean is normally distributed with mean 16.08 and standard deviation $\frac{.122}{3} = .041$.

(a) $P[\bar{X} < 16] = P[Z < \frac{16-16.08}{.041} = -1.95] = .0256$

(b) This has nothing to do with sampling, and refers to the probability that the parent variable takes values less than 16.0. $P[X < 16] = P[Z < \frac{16-16.08}{.122} = -.66] = .2546$

Problem 20

(a) Since the parent variable is normal, $\bar{X}$ is normally distributed with mean 3.5 and standard deviation .1. (Since the parent variable is inherently positive, it cannot be exactly normal; However we are assuming it is normal to a very good approximation. Since this is the case the sample mean, even for small samples, is also normally distributed to at least as good an approximation as the parent variable.)

(b) i $P[\bar{X} > 3.7] = P[Z > 2] = .0228$

ii $P[3.34 < \bar{X} < 3.66] = P[-1.6 < Z < 1.6] = .9452 - .0548 = .8904$

Problem 26. $\bar{X}$ is normally distributed with mean .05 and standard deviation .0015.

$$P[.048 < \bar{X} < .053] = P[-1.33 < Z < 2] = .9772 - .0918 = .8854$$

Problem 28.

(a) The mean is 3.7 days, and the standard deviation is .78 days.

(b) The total time for the transaction can take values between six and ten days inclusive. If Y is the total time, the distribution is calculated one value at a time as follows:

Y=6: This requires that both the original letter and the return receipt take only three days. Since each of these events individually has probability .5 and they are independent, $P[Y = 6] = .5^2 = .25$.

Y=7: Either the original letter takes three days and the return receipt four, or *vice versa*, so $P[Y = 7] = 2 \times .5 \times .3 = .3$

Y=8: This can happen three ways; each letter takes four days, the first takes three and the return five, or *vice versa*. Thus $P[Y = 8] = .3^2 + 2 \times .2 \times .5 = .29$

Y=9: This corresponds to four days followed by five or *vice versa* so $P[Y = 9] = 2 \times .3 \times .2 = .12$.

Y=10: Both the original and the return take five days. $P[Y = 10] = .2^2 = .04$

A check of the correctness of this computation is provided by the fact that the probabilities obtained add up to 1.

(c) Let X be the number of the letters taking five days. Then X is binomially distributed with $n = 100; \quad p = .2$. It follows that X has mean 20 and standard deviation 4. Thus

$$P[X > 25] \approx P[\hat{X} > 25.5] = P[Z > 1.375] = .0845$$