STAT 400 SUMMER II 2001 (PROFESSOR GREEN)
SOLUTIONS TO PROBLEMS DUE AUGUST 1

12.

(a) $\frac{1}{2}$
(b) $\frac{11}{16}$
(c) $\frac{81}{256}$
(d) $F'(x) = \frac{3}{64}(8-x)$ for $-2 \le x \le 2$ and 0 otherwise.
(e) This follows from (a).

14.

(a) The mean is 13.75 and the variance is 13.028.
(b) The cdf is 0 for $x < 7.5$, $\frac{x-7.5}{12.5}$ for $7.5 \le x \le 20$, and 1 for $x > 20$.

(c) .2, .4
(d) .772, 1.

18.

(a) $A + p(B - A)$.
(b) $E(X) = \frac{A+B}{2}$, $V(X) = \frac{(B-A)^2}{12}$, so that $\sigma = \frac{B-A}{\sqrt{12}}$.
(c) $\frac{B^{n+1} - A^{n+1}}{(n+1)(B-A)}$

20.

(a) The cdf is given by $F(y) = 0$ for $y < 0$, $F(y) = \frac{y^2}{50}$ for $0 \le y \le 5$, $F(y) = 1 - \frac{(10-y)^2}{50}$ for $5 \le y \le 10$, and $F(y) = 1$ for $y > 10$.
(b) For $p \le .5$, the $(100p)$th percentile is $5\sqrt{2p}$. For $p \ge .5$ it is $10 - 5\sqrt{2-2p}$.
(c) $E(Y) = 5$ and $V(Y) = \frac{25}{6}$, which are just twice the values for the uniformly distributed waiting time for a single bus.

24.

(a) $\frac{k\theta}{k-1}$
(b) If $k = 1$, the integral defining $E(X)$ diverges.

(c) If $k > 2$ the integral defining $E(X^2)$ converges to $\frac{k\theta^2}{k-2}$ and $E(X^2) - E(X)^2$ gives the desired result.

(d,e) In order for the integral defining $E(X^n)$ to converge, k must be strictly greater than n. In particular, the variance is not defined unless $k > 2$.