STAT 400 SUMMER II 2000 (PROFESSOR GREEN)
SOLUTIONS TO PROBLEMS DUE AUGUST 15

4. One should test $H_0 : \mu = 5$ against $H_a : \mu < 5$, and accept the water as safe only if the null hypothesis is rejected.

6. There are a number of reasonable ways of answering this question. Here is the one that seems most reasonable to me: If the mean is unsatisfactory, the manufacturer will have to overhaul his equipment or redesign his manufacuring procedures. Doing this unnecessarily is the consequence of a type I error for a two-sided test of $\mu = 40$. On the other hand, a type II error could lead to either excessive complaints or liability claims. There are presumably safe values, say 40.5 and 39.5, which will lead to neither many complaints nor serious liability. The sample size should be chosen large enough to make a type II error unlikely at either of these values. Because of the symmetry of a two-sided test, $\beta(40.5) = \beta(39.5)$.

10.

(a) $H_0 : \mu = 1300$ against $H_1 : \mu > 1300$.
(b) $\bar{X}$ is normally distributed with mean 1300 and standard deviation 13.4164. The probability of a type I error is about 1%.
(c) In this case, the distribution of the test statistic is still normal with the same standard deviation and mean 1350. The probability of a type II error is about 8.1%.
(d) The rejection region should be changed to $\bar{x} \geq 1322.07$. This will reduce the probability of a type II error at 1350 to about 1.9%
(e) $z \geq 2.33$

12.

(a) The parameter of interest is the average braking distance for the new design, which should be adopted only if the average braking distance is clearly lower than 120 ft. The relevant test is of $H_0 : \bar{X} = 120$ against $H_1 : \bar{X} < 120$. Actually, it might be better to define some minimum worthwhile improvement, say to 118 or 117 and test that on a one-sided basis.
(b) The appropriate region is R_2.

(c) Assuming the null hypothesis is $\mu = 120$, the significance level is about .2%. To reduce it to .1%, the rejection region would have to be $\bar{x} \leq 114.85$.
(d) The probability of a type II error in this case is upwards of 45%.
(e) About 1% and .2% respectively.