STAT 400 SUMMER II 2001 (PROFESSOR GREEN) SOLUTIONS TO PROBLEMS DUE AUGUST 9

4. In each case, the endpoints of the interval are $58.3 + d$ and $58.3 - d$, where d is the error bound. The values of d are given by (a) 1.176, (b) .588, (c) .773, and (d) .402. The answer to (e) is $n = 239$.

6. The confidence interval in part (a) is (8406.1, 8471.9). The interval for (b) is extended to (8404,8474).

8. As discussed in class, the more general interval is always longer than the symmetric one.

10. The discussion in class was in error. The standard deviation for an exponential variable is the same as the mean. This leads to a much simpler equation. The standard error is $\frac{\mu}{\sqrt{n}}$, leading to the inequality

$$-z_{\frac{\alpha}{2}}\frac{\mu}{\sqrt{n}} \leq \bar{x} - \mu \leq z_{\frac{\alpha}{2}}\frac{\mu}{\sqrt{n}},$$

leading in turn to the $1 - \alpha$ confidence interval

$$\frac{\bar{x}}{1 + \frac{z_{\frac{\alpha}{2}}}{\sqrt{n}}}, \frac{\bar{x}}{1 - \frac{z_{\frac{\alpha}{2}}}{\sqrt{n}}}.$$

If one expands the coefficients of $\bar{x}$ as a series in powers of $\frac{z_{\frac{\alpha}{2}}}{\sqrt{n}}$, and neglects the terms higher than first order, one obtains the usual confidence interval using $\bar{x}$ to estimate σ. However, the sample size is too small to permit neglecting these terms.

From the data of the problem, $\bar{x} = 4.2133$. The 95% confidence interval is (2.7976, 8.5302), and the 99% confidence interval is (2.5304, 12.5816). Since the mean and standard deviation are equal, the confidence intervals for the standard deviation are the same as for the mean.

12. The desired interval is (.7265,.8935).

18. The desired lower bound is 4.061.

22. The desired upper confidence bound (using the linear method and estimating the standard error by replacing the true proportion by the sample proportion) is 9.93%.

24. The desired confidence interval is (7.798, 8.543). There are no explicit assumptions on the actual distribution, but since the sample size is not extremely large, the results could be unreliable if the actual distribution were somewhat pathological.