STAT 400 SUMMER II 2001 (PROFESSOR GREEN)
SOLUTIONS TO PROBLEMS DUE JULY 24

4. For a five-digit zip-code, the possible values of X are integers between 0 and 5 inclusive. For example $X(20902) = 3$ and $X(49117) = 5$.

6. X can take any positive integer as a value. For example $X(L) = 1$, $X(AL) = X(RL) = 2$, and $X(AAL) = X(ARL) = X(RAL) = X(RRL) = 3$.

10.

(a) T takes integer values between 0 and 10 inclusive.
(b) Assuming the difference is always taken in the positive sense, X takes integer values between 0 and 6 inclusive.
(c) U takes values between 0 and 6 inclusive.
(d) Z takes values between 0 and 2 inclusive.

12. Only the middle function has probabilities summing to 1, and therefore only that one is a legal pmf.

14.

(a) $k = \frac{1}{15}$
(b) $\frac{2}{5}$
(c) $\frac{3}{5}$
(d) No; the probabilities would not sum to 1. However $p(y) = \frac{y^2}{55}$ could be such a pmf.

18. The values of M are 1 through 6 inclusive. The number of possible outcomes with maximum k is $2k - 1$, so that the pmf is given by $p(x) = \frac{2x-1}{36}$. The cdf is given by $F(x) = \frac{x^2}{36}$.

22.

(a) $p(2) = .2$
(b) $P(X > 3) = .33$
(c) $P(2 \leq X \leq 5) = .78$
(d) $P(2 < X < 5) = .53$

26. I will not list all the permutations of 1 through 4; however there are 24 of them. Notice first that X cannot take the value 3. Notice next that there is only one outcome in which X takes the value 4. When X takes the value 2, two students receive their own books and the other two are exchanged. This means that there are exactly 6 such outcomes, one for each pair of students receiving their own books. When $X = 1$, exactly one student receives his own book and the other three must be permuted cyclically. This can be done in two ways, so that the total number of outcomes in this case is 8. It follows that there are exactly 9 outcomes in which no student receives the correct book, and that the pmf is given by

$$p(0) = \frac{3}{8}, \quad p(1) = \frac{1}{3}, \quad p(2) = \frac{1}{4}, \quad p(4) = \frac{1}{24}.$$