STAT 400 SUMMER II 2000 (PROFESSOR GREEN)
SOLUTIONS TO PROBLEMS DUE JULY 26

44. The answers below are exact.
    (a) .12386304
    (b) .27869184
    (c) .63479808
    (d) .717570463519

46. These answers are from the binomial tables.
    (a) .873
    (b) .007
    (c) .716
    (d) .277
    (e) $E(X) = 1.25$; $\sigma = 1.09$.

54. \$40

60.
    (a) This is the probability that at most one individual with a reservation fails to appear, or $P(X \leq 1)$ for a binomial variable of type $(6, .2)$. The probability is .65536, by direct computation.
    (b) If we permitted a negative number of available places, the answer would be the number of places in the limousine minus the expected number of passengers. This is $4 - 4.8 = -.8$ This is incorrect because we have assumed respectively -1 and -2 places when five or six passengers appear. We can correct this by adding the probability that five passengers appear plus twice the probability that six passengers appear. This correction term is .917504, giving the expected number of available seats as .117504.
    (c) Let X be the number of passengers who appear. Then the pmf for X is given by

$$p(x) = .1b(x; 3, .8) + .2b(x; 4, .8) + .3b(x; 5, .8) + .4b(x; 6, .8),$$

where $b(x; , n, p)$ denotes the probability of the value x for a binomial distribution of type (n, p). Let Y be the number of

passengers who actually travel. Then Y has the same distribution as X for values between 0 and 3 inclusive, and $P(Y = 4) = P(X = 4) + P(X = 5) + P(X = 6)$.

62.

(a) $P(X = 2) = .4196$
(b) $P(X \leq 2) = .7133$
(c) $P(x \geq 2) = .7062$
(d) $E(X) = 2$; $V(X) = .8571$

66.

(a) The distribution is hypergeometric with $n = 6$, $M = 4$, and $N = 11$.
(b) 2.1818

68.

(a) The distribution is hypergeometric with $n = 10$, $M = 15$, and $N = 50$.
(b) The pmf is still hypergeometric with the same $n = 10$, $M = 150$ and $N = 500$. However this distribution is reasonably well approximated by a binomial distribution of type $(10, .3)$.
(c) The mean is 3 for both the exact and approximate distribution. The exact variance is 2.062, and the approximating variance is 2.1.

72. The total number of rolls is $5 + X$, where X is negative binomial of type $(5, \frac{1}{6})$. X has mean 25 and variance 150. $X + 5$ has the same variance and mean 30.