STAT 400 SUMMER II 2000 (PROFESSOR GREEN)
SOLUTIONS TO ASSIGNED PROBLEMS DUE JULY 31

2.

(a) .5
(b) .25
(c) .5
(d) .4

4.

(a) Integrate by substitution $u = \frac{x^2}{2\theta^2}$. The integral becomes $\int_0^\infty e^{-u}\,du = 1$.
(b) The same substitution yields

$$P(X \leq 200) = \int_0^2 e^{-u}\,du = 1 - e^{-2} = .8647.$$

$P(X < 200)$ is the same, while $P(X \geq 2) = .1353$.
(c) .4712
(d) $1 - e^{-\frac{x^2}{20{,}000}}$

6.

(a) The graph is a parabola with zeroes at 2 and 4 and a maximum at 3.
(b) $k = \frac{3}{4}$
(c) Since the curve is symmetric about $x = 3$, the probability that $X > 3)$ is .5.
(d) .3672
(e) .3125

8.

(a) The graph is an isoceles triangle with vertices at $(0,0)$, $(0,.2)$, and $(10,0)$.
(b) The area of the triangle is clearly 1.
(c) .18
(d) .92
(e) .74
(f) .4

10.

(b) The integral is

$$\int_{\theta}^{\infty} \frac{k\theta^k}{x^{k+1}}\,dx$$

and the antiderivative of the integrand is $-\frac{\theta^k}{x^k}$. It follows that the value of the integral is 1.

(c) $1 - \frac{\theta^k}{b^k}$.

(d) $\frac{\theta^k}{a^k} - \frac{\theta^k}{b^k}$.

58.

(a,b) $\mu = \sigma = 1$.
(c) $1 - e^{-4} = .9817$
(d) $e^{-2} - e^{-5} = .1286$

60. The relevant distribution is exponential with $\lambda = \frac{1}{25000}$. The cdf is given by $1 - e^{-\frac{t}{25000}}$.

(a) $e^{-.8} = .4493$, $1 - e^{-1.2} = .6988$, $e^{-.8} - e^{-1.2} = .1481$
(b) $e^{-3} = .0498$, $e^{-4} = .0183$.

62. The cumulative distribution function of each component, applied to t gives the probability that the time to failure is at most t, which is to say that the component will have failed by time t. For an exponentially distributed time to failure, the cdf is $1 - e^{-\lambda t}$. It follows that the probability that the i^{th} component of the system will still be operating at time t is $e^{-\lambda_i t}$. Since the components are in series, the system will still be operating at time t if and only if all the components are still operating. The probability for this is $e^{-\lambda t}$ where $\lambda = \sum_i \lambda_i$. It follows that the lifetime of the system is also exponentially distributed. In particular, if all the components have the same mean time to failure, then the mean time to failure of the system is the mean time of each component, divided by the number of components.

64. Make the substitution $u = \frac{x}{\beta}$.