EXAM 1: **NAME:**_____

1) (8pts) Prove or disprove the following: the product of a non-zero rational and an irrational is irrational.

2) (12pts) Show that $L_n = f_{n-1} + f_{n+1}$ where $L_n = L_{n-1} + L_{n-2}$, $L_1 = 1$ and $L_2 = 3$.

3) (10pts) Show if $a, b, c \in \mathbb{Z}$, $c \neq 0$ then $a|b$ **iff** $ac|bc$.

4) (10pts) Suppose $a, b, c \in \mathbb{Z}$ with $(a, b) = 1$. Show that if $c|(a + b)$ then $(c, a) = 1$.

5) (8pts) Use the extended Euclidean Algorithm to find the $gcd(198, 54)$ and write it as a linear combination of 198 and 54.

6) (12pts) Show if $a^3|b^2$ then $a|b$.

7) (8pts) Show if $a, b \in \mathbb{Z}$, $m, n \in \mathbb{Z}^+$ such that $n|m$ and $a \equiv b \ (mod\ m)$ then $a \equiv b \ (mod\ n)$.

8) (10pts) Show $x^2 \equiv 1 \ (mod\ p)$, with p an odd prime, has two incongruent solutions and list them.

9) (12pts) Find all the simultaneous solutions to $x \equiv 0 \pmod 4$, $x \equiv 2 \pmod 3$, and $x \equiv 2 \pmod 5$.

10) Using techniques from section 5.1 do the following.

a) (5pts) What is the highest power of 5 dividing $111,250$?

b) (5pts) What digit x makes $1x1051$ divisible by 11?