

## HOMEWORK 2

- 1) Let  $P$  : 7 is even and  $Q$  : 19 is prime. Determine whether the following are true or false. Justify each part.
 

|                        |                             |
|------------------------|-----------------------------|
| a) $P \vee Q$          | e) $Q \Rightarrow P$        |
| b) $P \wedge Q$        | f) $(\sim P) \Rightarrow Q$ |
| c) $(\sim P) \wedge Q$ | g) $P \Leftrightarrow Q$    |
| d) $P \Rightarrow Q$   | h) $(\sim P) \vee (\sim Q)$ |
- 2) Let  $P$  :  $\sqrt{3}$  is rational and  $Q$  :  $\frac{3}{2}$  is rational. Determine whether the following are true or false. Justify each part.
 

|                      |  |
|----------------------|--|
| a) $P \vee Q$        | e) $(\sim P) \Rightarrow (\sim Q)$     |
| b) $P \wedge Q$      | f) $(\sim Q) \Rightarrow (\sim P)$     |
| c) $P \Rightarrow Q$ | g) $P \Leftrightarrow Q$               |
| d) $Q \Rightarrow P$ | h) $(\sim P) \Leftrightarrow (\sim Q)$ |
- 3) In each of the following two open sentences  $P(x)$  and  $Q(x)$  are defined over a given domain  $S$ . Determine all  $x \in S$  for which  $P(x) \Rightarrow Q(x)$  is true.
  - a)  $P(x) : 5x + 3$  is prime;  $Q(x) : 7x + 1$  is prime;  $S = \{1, 2, 3, 4\}$
  - b)  $P(x) : x^2 = 9$ ;  $Q(x) : |x| = 3$ ;  $S = \{-3, -2, 0, 1, 3\}$
  - c)  $P(x) : x^2 \geq 4$ ;  $Q(x) : x \geq 2$ ;  $S = \mathbb{R}$
  - d)  $P(x) : x^2 \geq 4$ ;  $Q(x) : x \geq 2$ ;  $S = \mathbb{N}$
- 4) In each of the following two open sentences  $P(x, y)$  and  $Q(x, y)$  are defined over a given domain  $S$ . Determine all  $x \in S$  for which  $P(x, y) \Rightarrow Q(x, y)$  is true.
  - a)  $P(x, y) : x^2 - y^2 = 0$ ;  $Q(x, y) : x = y$ ;  $S = \{(-1, 1), (2, 3), (4, 4)\}$
  - b)  $P(x, y) : |x| = |y|$ ;  $Q(x, y) : x = y$ ;  $S = \{(-2, 2), (3, 3), (-1, 2)\}$
- 5) Repeat homework problem three with  $P(x) \Leftrightarrow Q(x)$  instead of  $P(x) \Rightarrow Q(x)$ .
- 6) Repeat homework problem four with  $P(x, y) \Leftrightarrow Q(x, y)$  instead of  $P(x, y) \Rightarrow Q(x, y)$ .
- 7) Give a truth table for each of the following.
  - a)  $(P \vee Q) \Rightarrow P$
  - b)  $[P \Rightarrow (Q \vee R)] \equiv [(\sim Q) \Rightarrow ((\sim P) \vee R)]$
  - c)  $[(P \vee Q) \Rightarrow R] \equiv [(P \Rightarrow Q) \wedge (Q \Rightarrow R)]$
- 8) Negate each of the following open sentences. You should remove 'not' from your sentences.
 

|                                     |                                      |
|-------------------------------------|--------------------------------------|
| a) $a = 0$ and $b = 0$              | c) If $a = 0$ , then $b = 0$ .       |
| b) Either $x$ is odd or $y$ is odd. | d) If $x$ is even, then $y$ is even. |
- 9) Determine whether the following are true or false. Justify each part.
 

|   |  |
|---|--|
| a) $\forall x \in \mathbb{R}, \sqrt{x^2} = x$ | e) $\exists x \in \mathbb{Q}, \frac{1}{x^2} = \frac{1}{2}$ |
| b) $\exists x \in \mathbb{R}, x^2 - x = 0$    | f) $\forall x \in \mathbb{Q}, x^2 > 0$                     |
| c) $\forall n \in \mathbb{N}, n + 1 \geq 2$   | g) $\exists x, y \in \mathbb{R}, x + y + 1 = 2$            |
| d) $\exists x \in \mathbb{Q}, 27 - 3x^2 = 0$  | h) $\forall x, y \in \mathbb{R}, x + y + 1 = 2$            |
- 10) Negate each of the following quantified statements. You should remove 'not' from your sentences.
 

|   |  |
|---|--|
| a) There exists a real number $x$ , such that $x^2 < 0$ . | c) $\forall n \in \mathbb{N}, 27 - 3n^2 = 0$   |
| b) For all integers $n$ , $n^2 + 4 = 3$ .                 | d) $\exists x \in \mathbb{R}, x^2 + x + 1 = 0$ |
- 11) Homework problem 2.61 from the text.