HOMEWORK 3

(1) 1) Let $n \in \mathbb{N}$. Prove if $n^2 < 1$, then $n^2 + 3n + 4 \leq 0$.

2) Let $x \in \mathbb{Z}$. Prove if $\frac{x}{3} - x = 0$, then $(x+1)^2 \geq 0$.

(2) Direct Proof:

3) Prove if n is odd, then $3n - 6$ is odd.

4) Prove if n is even, then $7n - 2$ is even.

5) Prove if x and y have the same parity, then $x + y$ is even.

6) Prove if $n \in \mathbb{Z}$, then $n^2 + n + 1$ is odd.

(3) Contrapositive Proof:

7) Prove if $5x + 1$ is odd, then x is even.

8) Prove if $3x - 7$ is even, then x is odd.

9) Prove if $x + y$ is even, then x and y have the same parity.

(4) You Choose:

10) Prove xy and $x + y$ are both even if and only if x and y are both even.

11) Prove for every two distinct real numbers a and b, either $\frac{a+b}{2} > a$ or $\frac{a+b}{2} > b$.

12) State the problem that numbers 5 and 9 give together.