HOMEWORK 4

1) Let $a, b, c \in \mathbb{Z}$ with $a \neq 0$.
   a) Prove: if $a \not| bc$, then $a \not| b$ and $a \not| c$.
   b) Does $a|bc$ imply $a|b$ or $a|c$?

2) Let $m \in \mathbb{Z}$
   a) Give a direct proof of the following: If $5|m$, then $5|m^2$.
   b) State the contrapositive of part a.
   c) Five a direct proof of the following: If $5 \not| m$, then $5 \not| m^2$.
   d) State the contrapositive of part c.
   e) State the conjunction of a and c using "if and only if".

3) Let $a, b \in \mathbb{Z}$. Prove $3|ab$ if and only if $3|a$ or $3|b$.

4) Let $x \in \mathbb{R}$. If $x^3 - 3x^2 + 2x = 6$, then $x = 3$.

5) Let $x, y \in \mathbb{R}$. Prove that $|xy| = |x| \cdot |y|$.

6) Let $x \in \mathbb{R}$. If $x^7 + 3x^3 + x \geq 2x^6 + x^2 + 1$, then $x > 0$.

7) Prove that every two positive real numbers a and b, $\frac{a}{b} + \frac{b}{a} \geq 2$.

8) a) Give an example of three sets A, B, C such that $A \cap B = A \cap C$, but $B \neq C$.
   b) Give an example of three sets A, B, C such that $A \cup B = A \cup C$, but $B \neq C$.
   c) Let A, B, C be sets. Prove that if $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then $B = C$.

9) Prove Theorem 4.21($2a$).

10) Prove Theorem 4.21($4b$).

11) Let A, B be sets. Prove that if $A \cap B = \phi$, then $A = (A \cup B) - B$.