HOMEWORK

1) Disprove: If $a, b \in \mathbb{R}$, then $\log \frac{a}{b} = \frac{\log a}{\log b}$.

2) Disprove: For all $x, y \in \mathbb{R}$, $(x - y)^2 > 0$.

3) Prove by contradiction: Let $s \in \mathbb{I}$ and $r \in \mathbb{Q}$ with $r \neq 0$. Prove $\frac{s}{r} \in \mathbb{I}$.

4) Prove by contradiction: Let x be even and y be odd. Prove $x + y$ is odd.

5) Prove by contradiction: Let $m \in \mathbb{Z}^+$ and s be an odd integer. Assume $m = 2s$. Prove there do not exist $x, y \in \mathbb{Z}^+$ with $x^2 - y^2 = m$.

6) Prove by contradiction: Let $a, b \in \mathbb{Z}$. Prove if $a \geq 3$, then $a \nmid b$ or $a \nmid (b+1)$ or $a \nmid (b+2)$.

7) Prove by contradiction:
   a) Let a_1, a_2 be odd integers greater than one. Prove that if $n = a_1 \cdot a_2 + 2$, then $a_1 \nmid n$ and $a_2 \nmid n$.

   b) Extend this to a product of r odd integers greater than one. So, prove: if $a_1, a_2, \cdots, a_r$ are all odd integers greater than one and $n = a_1 \cdot a_2 \cdots a_r + 2$, then $a_i \nmid n$ for $1 \leq i \leq r$.

8) Prove by contradiction: That $\sqrt{5}$ is irrational.

9) Give a correct proof of the result in problem 5.46.

10) Show that there exist no nonzero real numbers a, b such that $\sqrt{a^2 + b^2} = \sqrt[4]{a^4 + b^4}$.

11) a) Show that there exists a solution to $x^3 - 2x^2 + 2 = 0$ in the interval $[-1, 1]$.

    b) Show the solution in part a is unique.

12) Disprove: There exists an $n \in \mathbb{Z}$ with $n^2 + n$ odd.

13) Disprove: There is a real number x such that $x^6 + x^2 + 1 = 2x^4$.