HOMEWORK 6

Regular Induction:

1) Prove by induction that $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$ for all natural numbers n.

2) Find a formula for the sum $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \cdots + \frac{1}{(2n-1)(2n+1)}$. Verify the formula is correct by induction.

3) Let $b, t \in \mathbb{R}$ with $t \neq 1$. Prove $b + bt + bt^2 + \cdots + bt^{k-1} = \frac{b(t^k - 1)}{t-1}$ for all positive integers k.

4) a) Find a formula for the sum $1 + 3 + 5 + \cdots + 2n - 1$ and verify it is correct by induction.

   b) Use Result 6.3 (or 6.4) from the text to find a formula for $2+4+6+\cdots+2n$.

   c) Use parts a and b to find a formula for the sum $1 - 2 + 3 - 4 + \cdots + (2n - 1) - 2n$.

   d) Use parts a and b to find a formula for the sum $-1 + 2 - 3 + 4 - \cdots - (2n - 1) + 2n$.

General Induction:

5) Prove with induction that $6|(m^3 - m)$ for all $m \in \mathbb{Z}$. [Hint: Start with $m \geq 0$ and then prove if $6|(m^3 - m)$, then $6|((-m)^3 - (-m))$.]

6) Prove by induction $1 + 2 + 2^2 + \cdots + 2^n = s^{n+1} - 1$ for $n \geq 0$.

7) Find a formula for the sum $1 + \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^n}$. Verify by induction that the formula is correct for all non-negative integers.