HOMEWORK

1) For every nonnegative integer n, there exists a nonnegative integer k such that $k < n$.
   a) Write the statement in symbols.
   b) Write the negation of the statement.
   c) Prove the statement is false.
2) For all sets A and B, $(A \cup B) - B = A$.
   a) Write the statement in symbols.
   b) Write the negation of the statement.
   c) Prove the statement is false.
3) For every set $A \neq \phi$, there exists a set B such that $A \cap B = \phi$.
   a) Write the statement in symbols.
   b) Write the negation of the statement.
   c) Prove the statement is true.
4) There exists an odd integer, the sum of whose digits is odd and the product of whose digits is odd.
   a) Write the statement in symbols.
   b) Write the negation of the statement.
   c) Prove the statement is true.
5) For all $x, y \in \mathbb{R}$, if $x^3 < y^3$ then $x < y$.
   a) Write the statement in symbols.
   b) Write the negation of the statement.
   c) Prove the statement is true. [You may not use calculus.]
6) For all $x, y \in \mathbb{R}$, if $x^4 < y^4$ then $x < y$.
   a) Write the statement in symbols.
   b) Write the negation of the statement.
   c) Prove the statement is false. [You may not use calculus.]

Prove or Disprove each of the following:

7) a) There exists an odd integer, the sum of whose digits is odd and the product of whose digits is even.
   b) There exists an even integer, the sum of whose digits is odd and the product of whose digits is even.
   c) There exists an even integer, the sum of whose digits is even and the product of whose digits is odd.
8) Let A be a set. If, for all sets B, $A \cap B = \phi$, then $A = \phi$.
9) For every positive irrational number b, there exists a rational number a, such that $0 < a < b$.
10) Let A be a set. If, for all sets B, $A \cup B \neq \phi$, then $A \neq \phi$.
11) There exist positive integers x and y such that $x^2 - y^2 = 101$.
12) Every rational number is the sum of two irrational numbers.
13) Let A, B be sets. If $A \subseteq (A \cup B) - B$, then $A \cap B = \phi$.
14) Let $a, b, c \in \mathbb{Z}$. Then at least one of the sums: $a + b$, $a + c$, $b + c$, is even.
15) There exists a real solution to the equation $x^6 + 2x^4 = -1$.
16) There exists a real solution to the equation $x^5 + x = -1$.
17) Let $r, s \in \mathbb{Q}$ with $r < s$. There exists a rational number a with $r < a < s$.
18) For all natural numbers n, $n^2 - n + 43$ is prime.
19) Hwk 7.45.