HOMEWORK 8

1) Determine if the following relations are functions from the domain that is given to the codomain $\mathbb{R}$.
   a) Domain: $D_1 = [0, \infty)$; Relation: $R_1 = \{(x, y) : x \in \mathbb{D}_1, y^2 + 1 = x\}$
   b) Domain: $D_2 = \mathbb{R}$; Relation: $R_2 = \{(x, y) : x \in \mathbb{D}_1, y = 3x + 2\}$
   c) Domain: $D_3 = \mathbb{R}$; Relation: $R_3 = \{(x, y) : x \in \mathbb{D}_1, (y + x)^2 = 1\}$

2) For each of the following functions find the largest possible domain that is a subset of $\mathbb{R}$. Given this domain, find the range of each function.
   a) $f_1(x) = \sqrt{3x + 1}$
   b) $f_2(x) = \frac{2x+1}{x-4}$
   c) $f_3(x) = 2x^2 + x + 1$

3) Is the function $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = 3n + 1$ injective and/or surjective?

4) Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = x^2 + x + 1$.
   a) Show that f is not injective.
   b) Find all pairs r_1, r_2 of real numbers such that $f(r_1) = f(r_2)$.
   c) Show that f is not surjective.
   d) Find the set of real numbers S such that if $s \in S$, then there does not exist an $x \in \mathbb{R}$ with $f(x) = s$.

5) Prove all quadratic functions on $\mathbb{R}$ of the form $f(x) = ax^2 + bx + c$ with $a \neq 0$ and $a, b, c \in \mathbb{R}$ are neither one-one, nor onto. [Hint: Consider $b = 0$ and $b \neq 0$ separately and the quadratic formula may be helpful.]

6) Let $A = [0, 1] \subseteq \mathbb{R}$. Give two examples of functions from $A \to A$ that are not the identity, but are bijective.

7) Define $f : \mathbb{R} - \{-2\} \to \mathbb{R} - \{3\}$ by $f(x) = \frac{3x-1}{x+2}$. Prove f is bijective.

8) Define $f : \mathbb{R} - \{1\} \to \mathbb{R} - \{-2\}$ by $f(x) = \frac{-2x+1}{x-1}$. Prove f is bijective.