HOMEWORK 1

1) Write each of the following sets by listing the elements with braces.
   a) $A = \{x \in \mathbb{R} : x^2 = 9\}$
   b) $B = \{n \in \mathbb{Z} : n^3 < 36\}$
   c) $C = \{x \in \mathbb{R} : x^2 + 4 = 0\}$
   d) $D = \{n \in \mathbb{N} : |n| \leq 11\}$
2) Write each of the following sets in the form $\{x \in S : p(x)\}$ where $p(x)$ is a property on x and $S = \{-7, -6, -1, 0, 1, 3\}$.
   a) $A = \{-1, 0, 1\}$
   b) $B = \{-7, -6, -1\}$
   c) $C = \{-1, 1\}$
   d) $D = \{-7, -6, 3\}$
3) Give examples of sets A, B, and C such that the following hold. In each example it must be clear to the grader that you understand the properties required.
   a) $A \in B$, $B \in C$ and $A \subseteq C$
   b) $A \subset B$, $B \in C$, and $A \notin C$
   c) $A \in B$, $B \subseteq C$ and $A \not\subseteq C$
   d) $B \in A$, $B \subset C$, and $A \cap C \neq \phi$
4) Let $A = \{\phi, \{\phi\}, \{\{\phi\}\}\}$. (For $d - g$ determine the set indicated.)
   a) List the elements of A.
   b) Determine $|A|$.
   c) List all possible subsets of A.
   d) $\phi \cap A$
   e) $\{\phi\} \cap A$
   f) $\{\phi\} \cup A$
   g) $\{\phi, \{\{\phi\}\}\} \cup A$
5) Give an example of a universal set U, two sets A and B, and a Venn diagram such that $|A \cap B| = |A - B| = |B - A| = |\overline{A \cup B}| = 2$.
6) Let A and B be sets in some unknown universal set U. Suppose $\bar{A} = \{3, 7, 9\}$, $A - B = \{1, 2\}$, $B - A = \{7\}$, and $A \cap B = \{5, 8\}$. Determine A, B, and U.
7) Which of the following are true? Explain each of your answers.
   a) $\{1, 2, 3\} = \{2, 3, 1\}$
   b) $\phi \in \phi$
   c) $\phi \in \{1, 2, 3\}$
   d) $\phi \subseteq \phi$
   e) $3 \subseteq \{3\}$
   f) $\phi = \{\phi\}$
   g) $\phi \subset \{\phi\}$
8) Negate each of the following. You should remove 'not' from your sentences.
   a) π is rational.
   b) Five is not a positive number.
   c) 17 is composite.
   d) Six is prime.

9) Let P : 4 is odd and Q : 7 is prime. Determine whether the following are true or false. Justify each part.
   a) $P \vee Q$
   b) $P \wedge Q$
   c) $(\sim P) \wedge Q$
   d) $P \Rightarrow Q$
   e) $Q \Rightarrow P$
   f) $(\sim P) \Rightarrow Q$
   g) $P \Leftrightarrow Q$
   h) $(\sim P) \vee (\sim Q)$

10) Let $P : \sqrt{8}$ is rational and $Q : \frac{7}{3}$ is rational. Determine whether the following are true or false. Justify each part.
   a) $P \vee Q$
   b) $P \wedge Q$
   c) $P \Rightarrow Q$
   d) $Q \Rightarrow P$
   e) $(\sim P) \Rightarrow (\sim Q)$
   f) $(\sim Q) \Rightarrow (\sim P)$
   g) $P \Leftrightarrow Q$
   h) $(\sim P) \Leftrightarrow (\sim Q)$

11) In each of the following two open sentences $P(x)$ and $Q(x)$ are defined over a given domain S. Determine all $x \in S$ for which $P(x) \Rightarrow Q(x)$ is true.
   a) $P(x) : 3x + 1$ is prime; $Q(x) : 2x + 1$ is prime; $S = \{1, 2, 3, 4\}$
   b) $P(x) : x^2 = 4$; $Q(x) : |x| = 2$; $S = \{-3, -2, 0, 1, 3\}$
   c) $P(x) : x^2 \geq 9$; $Q(x) : x \geq 3$; $S = \mathbb{R}$
   d) $P(x) : x^2 \geq 9$; $Q(x) : x \geq 3$; $S = \mathbb{N}$

12) In each of the following two open sentences $P(x, y)$ and $Q(x, y)$ are defined over a given domain S. Determine all $x \in S$ for which $P(x, y) \Rightarrow Q(x, y)$ is true.
   a) $P(x, y) : x^2 - y^2 = 0$; $Q(x, y) : x = y$; $S = \{(1, -1), (3, 2), (4, 4)\}$
   b) $P(x, y) : |x| = |y|$; $Q(x, y) : x = y$; $S = \{(-3, 3), (2, 2), (-1, 2)\}$

13) Repeat homework problem eleven with $P(x) \Leftrightarrow Q(x)$ instead of $P(x) \Rightarrow Q(x)$.

14) Repeat homework problem twelve with $P(x, y) \Leftrightarrow Q(x, y)$ instead of $P(x, y) \Rightarrow Q(x, y)$.