

HOMEWORK 12

- 1) True or false. Justify your answers.
 - a) A subsequence of a bounded sequence is bounded.
 - b) A subsequence of a monotone sequence is monotone.
 - c) A subsequence of a convergent sequence is convergent.
 - d) A sequence converges if it has a convergent subsequence.
 - e) Every sequence in the interval $(0, 1)$ has a convergent subsequence.
 - f) Every sequence in the interval $(0, 1)$ has a subsequence that converges to a point in $(0, 1)$.
 - g) If a sequence of non-negative real numbers has a limit, it is also non-negative.
- 2) Let $a_1 = 1$ and $a_n = \sqrt{3a_{n-1} + 4}$ for $n \geq 2$. Prove $a_n < 4$ for all $n \in \mathbb{N}$.
- 3) Use monotone convergence to prove the following converge and then find the limit using properties of subsequences.
 - a) $s_1 = 1$ and $s_n = \frac{1}{4}(s_{n-1} + 5)$ for $n \geq 2$
 - b) $s_1 = 2$ and $s_n = \frac{1}{4}(s_{n-1} + 5)$ for $n \geq 2$
- 4) Fitzpatrick Section 2.4 number 7.
- 5) Fitzpatrick Section 2.4 number 8.
- 6) Fitzpatrick Section 2.4 number 9.