HOMEWORK 2

1) Give a truth table for each of the following. Please note if the statement is a tautology or contradiction or neither.
   a) $P \Rightarrow (P \wedge Q)$
   b) $[P \Rightarrow (R \vee Q)] \equiv [(\sim R) \Rightarrow ((\sim P) \vee Q)]$
   c) $[(P \vee Q) \Rightarrow R] \equiv [(P \Rightarrow Q) \wedge (Q \Rightarrow R)]$

2) Negate each of the following open sentences. You should remove 'not' from your sentences.
   a) $x = 0$ and $y = 0$
   b) Either a is even or b is even.
   c) If $s = 0$, then $t = 0$.
   d) If x is odd, then y is odd.

3) Determine whether the following are true or false. Justify each part.
   a) $\forall n \in \mathbb{N}, n + 1 \geq 2$
   b) $\exists x \in \mathbb{Q}, 27 - 3x^2 = 0$
   c) $\forall x \in \mathbb{R}, \sqrt{x^2} = x$
   d) $\exists x \in \mathbb{R}, x^2 - x = 0$
   e) $\exists x, y \in \mathbb{R}, x + y + 1 = 2$
   f) $\forall x, y \in \mathbb{R}, x + y + 1 = 2$
   g) $\exists x \in \mathbb{Q}, \frac{1}{x^2} = \frac{1}{2}$
   h) $\forall x \in \mathbb{Q}, x^2 > 0$

4) Negate each of the following quantified statements. You should remove 'not' from your sentences.
   a) There exists a real number y such that $y^4 < 0$.
   b) For all integers n, if $n^2 + 4 > 4$, then $n \neq 0$.
   c) $\forall n \in \mathbb{N}, 27 - n^3 > 0$
   d) $\exists x \in \mathbb{R}, x^3 + x^2 + x + 1 = 0$

5) Let $n \in \mathbb{N}$. Prove: If $n + 3 < 0$, then $n^2 + 1 > 11$.

6) Let $x \in \mathbb{Z}$. Prove: If $x + 17 = 11$, then $x^2 + 1 < 0$.

7) Prove directly:
   a) For $n \in \mathbb{Z}$, if n is even, then $7n + 3$ is odd.
   b) For $n \in \mathbb{Z}$, if n is odd, then $2n - 5$ is even.

8) Prove by contrapositive:
   a) For $n \in \mathbb{Z}$, if $3n + 1$ is odd, then n is even.
   b) For $n \in \mathbb{Z}$, if $3n - 8$ is odd, then n is odd.