HOMEWORK 3

1) Prove if $n \in \mathbb{Z}$ then $n^2 - n$ is even.

2) Let $x, y \in \mathbb{Z}$. Prove the sum $x + y$ is odd if and only if x and y have opposite parity.

3) Let $x, y \in \mathbb{Z}$.
   a) Prove if x and y are odd, then xy is odd.
   b) Prove if xy is odd, then x and y are odd.
   c) Together what statement do parts a and b give?

4) Let $x, y, z \in \mathbb{Z}$ with $x \neq 0$.
   a) Prove if $x \nmid yz$, then $x \nmid y$ and $x \nmid z$
   b) Is the statement "if $x|yz$, then $x|y$ or $x|z$" true or false?

5) Let $x, y \in \mathbb{Z}$. Prove if $3 \nmid x$ and $3 \nmid y$, then $3|(x^2 - y^2)$.

6) Prove if $5|2a$, then $5|a$.

7) Let $x, y \in \mathbb{R}$. Prove if $x^2 - 2x = y^2 - 2y$ and $x \neq y$, then $x = 2 - y$.

8) Let $x \in \mathbb{R}$. Prove if $2x^6 + x^2 + 1 \leq x^9 + 7x^3 + 2x$ then $x > 0$.

9) Let $x, y \in \mathbb{R}$. Prove if $x < 0$, then $x^3 - x^2y \leq x^2y - xy^2$.

10) Prove by contradiction that $\sqrt{3}$ is irrational.

11) Prove that the product of a nonzero rational and an irrational is irrational.

12) a) Disprove: If $a, b \in \mathbb{R}$, then $\log(a + b) = \log a + \log b$.
   b) Prove there exists a real solution to $x^3 + x - 1 = 0$ between $x = 0$ and $x = 1$.
   c) Disprove: There exists a real number x such that $x^4 + 3x^2 + 1 = 0$.