HOMEWORK

For each problem 1 through 5 do the following:

a) Write the statement in symbols.
b) State the negation of the statement (in symbols or words).
c) State the contrapositive of the statement (in symbols or words).

1) Let A, B be sets. If $A \cap B \neq \phi$, then $A \not\subseteq (A \cup B) - B$, .
2) Let A be a set. If, for all sets B, $A \cap B = \phi$, then $A = \phi$.
3) For all $x, y \in \mathbb{R}$, if $x^4 < y^4$ then $x < y$.
4) For all $x, y \in \mathbb{R}$, if $x^3 < y^3$ then $x < y$.
5) For every set $A \neq \phi$, there exists a set B such that $A \cap B = \phi$.

Prove or Disprove each of the following:

6) The statement in number one.
7) The statement in number two.
8) The statement in number three.
9) The statement in number four.
10) The statement in number five.
11)
    a) There exists an odd integer, the sum of whose digits is odd and the product of whose digits is even.
    b) There exists an even integer, the sum of whose digits is odd and the product of whose digits is even.
    c) There exists an even integer, the sum of whose digits is even and the product of whose digits is odd.
    d) There exists an odd integer, the sum of whose digits is odd and the product of whose digits is odd.
12) Let A be a set. If, for all sets B, $A \cup B \neq \phi$, then $A \neq \phi$.
13) Every rational number is the sum of two irrational numbers.
14) There exists a real solution to the equation $2x^6 + x^2 = -1$.
15) There exists a real solution to the equation $x^5 + x^3 = -1$.
16) Let $r, s \in \mathbb{Q}$ with $r < s$. There exists a rational number a with $r < a < s$.
17) For every positive integer n, $n^2 - n + 11$ is prime.
18) There exist positive integers x and y such that $x^2 - y^2 = 51$.
19) Let $x, y, z \in \mathbb{Z}$. If xy, xz and yz are even, then x, y and z are even.