HOMEWORK 7

1) Let $A = \{1, 2, 3\}$ and $B = \{a, b, c, d\}$. Define a relation R from A to B that contains exactly three elements, but is not a function from A to B. Describe why R is not a function (you should have two reasons).
2) For each part, using the given set A and relation R, determine if R is a function from A to $\mathbb{R}$.
   a) $A = \mathbb{R}$ and $R = \{(x, y) : 2y = 3x + 5\}$.
   b) $A = \mathbb{R}^+$ and $R = \{(x, y) : (x - y)^2 = 3\}$.
   c) $A = \mathbb{R}_{\geq 0}$ and $R = \{(x, y) : \sqrt{x} = y\}$.
3) In each part, determine the domain and range (both subsets of $\mathbb{R}$) for the given function.
   a) $f(x) = x^2 + 32$
   b) $g(x) = \sqrt{2x + 7}$
   c) $h(x) = \frac{2x}{x-9}$
4) Give an example of two sets C and D and functions $g : C \to D$ and $h : D \to C$ such that g is one-to-one, but not onto, and h is onto, but not one-to-one.
5) In each part, determine whether the give function is injective and/or surjective.
   a) $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = 2x + 1$
   b) $g : \mathbb{Z} \to \mathbb{Z}$ defined by $g(x) = x - 7$
   c) $h : \mathbb{R} \to \mathbb{R}$ defined by $h(x) = x^2$
6) Let $a, b \in \mathbb{R}$ with $b \neq 0$. Prove the function $f : \mathbb{R} - [a] \to \mathbb{R} - [b]$ defined by $f(x) = \frac{bx}{x-a}$ is bijective.
7) Define three different bijective functions from the interval $[0, 1]$ to itself.

For the rest of the problems: Let A, B, C be non-empty sets and $f : A \to B$ and $g : B \to C$ be functions.

8) a) Prove: If $g \circ f$ is injective, then f is injective.
   b) Disprove: If $g \circ f$ is injective, then g is injective.
   c) Prove or disprove: If $g \circ f$ is surjective, then f is surjective.
   d) Prove or disprove: If $g \circ f$ is surjective, then g is surjective.
9) Prove or disprove:
   a) If f is injective, then $g \circ f$ is injective.
   b) If f is surjective, then $g \circ f$ is surjective.
   c) If g is injective, then $g \circ f$ is injective.
   d) If g is surjective, then $g \circ f$ is surjective.
10) Prove or disprove:
   a) There exists a function f and g such that f is not injective, but $g \circ f$ is injective.
   b) There exists a function f and g such that g is not injective, but $g \circ f$ is injective.
   c) There exists a function f and g such that f is not surjective, but $g \circ f$ is surjective.
   d) There exists a function f and g such that g is not surjective, but $g \circ f$ is surjective.
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12) **More difficult than the others** Let $g \circ f$ be bijective. Prove:
   a) If f is onto, then g is one-to-one.
   b) If g is one-to-one, then f is onto.