

## HOMEWORK 1

- 1) In Chapter 0 under the heading “Using Symbols” state numbers 3 and 8 in your own words.
- 2) In Chapter 0 under the heading “Common Words and Phrases” state numbers 3 and 6 in your own words.
- 3) Write each of the following sets by listing the elements with braces.
  - a)  $A = \{x \in \mathbb{R} : x^3 = 8\}$
  - b)  $B = \{n \in \mathbb{Z} : n^2 < 6\}$
  - c)  $C = \{x \in \mathbb{R} : x^4 + 1 = 0\}$
  - d)  $D = \{n \in \mathbb{N} : |n| \leq 3\}$
- 4) Write each of the following sets in the form  $\{x \in S : p(x)\}$  where  $p(x)$  is a property on  $x$  and  $S = \{-5, -3, -2, 0, 2, 3\}$ .
  - a)  $A = \{-2, 0, 2\}$
  - b)  $B = \{-5, -3, -2\}$
  - c)  $C = \{-2, 2\}$
  - d)  $D = \{-5, -3, 3\}$
- 5) Give examples of sets  $A, B,$  and  $C$  such that the following hold. In each example it must be clear to the grader that you understand the properties required.
  - a)  $A \subset B$  and  $A \in C$
  - b)  $A \in B, A \subseteq C$  and  $B \cap C \neq \phi$
  - c)  $A \subseteq C, B \subset C,$  and  $A \cap B = \phi$
  - d)  $|A \cap B| = |A - B| = |B - A| = |\overline{A \cup B}| = 3$  and  $C = U$  the universal set.
- 6) Let  $A = \{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$ . (For  $d - g$  determine the set indicated.)
  - a) Determine  $|A|$ .
  - b) List the elements of  $A$ .
  - c) List all possible subsets of  $A$ .
  - d)  $\phi \cap A$
  - e)  $\{\{\phi\}\} \cap A$
  - f)  $\{\phi\} \cup A$
  - g)  $\{\phi, \{\{\phi\}\}\} \cup A$
- 7) Let  $U = \{e, f, g\}$  be the universal set, and let  $A = \{e, f\}, B = \{f, g\},$  and  $C = \{e, g\}$ . Determine the following:
  - a)  $(A \cup B) - (B \cap B)$
  - b)  $\overline{A}$
  - c)  $\overline{(B \cup C)}$
  - d)  $A \times B$
- 8) Which of the following are statements? State whether the statements are True or False.
  - a) 6 is odd.
  - b)  $7 - 2 = 8$
  - c)  $3x + 7 = 1$
  - d) multiply 4 and 10

9) Which of the following are true? Explain each of your answers.

- a)  $2 \subseteq \{2\}$
- b)  $\phi \in \{1\}$
- c)  $\phi \in \phi$
- d)  $\phi \subset \{\phi\}$

10) Negate each of the following. You should remove 'not' from your sentences.

- a)  $\pi$  is not rational.
- b) Zero is a negative number.
- c) Seven is odd.
- d) 22 is composite.

11) Let  $P$  : 4 is odd and  $Q$  : 7 is prime. Determine whether the following are true or false. Justify each part.

- a)  $P \vee Q$
- b)  $P \wedge Q$
- c)  $(\sim P) \wedge Q$
- d)  $P \Rightarrow Q$
- e)  $Q \Rightarrow P$
- f)  $(\sim P) \Rightarrow Q$
- g)  $P \Leftrightarrow Q$
- h)  $(\sim P) \vee (\sim Q)$

12) Let  $P$  :  $\sqrt{3}$  is rational and  $Q$  :  $\frac{4}{3}$  is rational. Determine whether the following are true or false. Justify each part.

- a)  $P \vee Q$
- b)  $P \wedge Q$
- c)  $P \Rightarrow Q$
- d)  $Q \Rightarrow P$
- e)  $(\sim P) \Rightarrow (\sim Q)$
- f)  $(\sim Q) \Rightarrow (\sim P)$
- g)  $P \Leftrightarrow Q$
- h)  $(\sim P) \Leftrightarrow (\sim Q)$

13) In each of the following two open sentences  $P(x)$  and  $Q(x)$  are defined over a given domain  $S$ . Determine all  $x \in S$  for which  $P(x) \Rightarrow Q(x)$  is true.

- a)  $P(x)$  :  $3x + 1$  is prime;  $Q(x)$  :  $2x + 1$  is prime;  $S = \{1, 2, 3, 4\}$
- b)  $P(x)$  :  $x^2 = 9$ ;  $Q(x)$  :  $|x| = 3$ ;  $S = \{-3, -2, 0, 1\}$
- c)  $P(x)$  :  $x^2 \geq 4$ ;  $Q(x)$  :  $x \geq 2$ ;  $S = \mathbb{R}$
- d)  $P(x)$  :  $x^2 \geq 4$ ;  $Q(x)$  :  $x \geq 2$ ;  $S = \mathbb{N}$

14) In each of the following two open sentences  $P(x, y)$  and  $Q(x, y)$  are defined over a given domain  $S$ . Determine all  $(x, y) \in S$  for which  $P(x, y) \Rightarrow Q(x, y)$  is true.

- a)  $P(x, y)$  :  $x^2 - y^2 = 0$ ;  $Q(x, y)$  :  $x = y$ ;  $S = \{(1, -1), (3, 2), (4, 4)\}$
- b)  $P(x, y)$  :  $|x| = |y|$ ;  $Q(x, y)$  :  $x = y$ ;  $S = \{(-3, 3), (2, 2), (-1, 2)\}$

15) Repeat homework problem 13 with  $P(x) \Leftrightarrow Q(x)$  instead of  $P(x) \Rightarrow Q(x)$ .

16) Repeat homework problem 14 with  $P(x, y) \Leftrightarrow Q(x, y)$  instead of  $P(x, y) \Rightarrow Q(x, y)$ .

- 17) Give truth tables for the following: Label each as a Tautology, Contradiction, or Neither.
- $P \wedge (Q \Rightarrow (\sim P))$
  - $(P \wedge Q) \Leftrightarrow P$  (what is this logically equivalent to?)
- 18) Verify the pairs of statements are logically equivalent.
- $P \Rightarrow (Q \vee R)$  and  $(\sim Q) \Rightarrow ((\sim P) \vee R)$
  - $(P \wedge Q) \Rightarrow R$  and  $(P \wedge (\sim R)) \Rightarrow (\sim Q)$
- 19) Negate the following: (whenever possible remove the 'not')
- Either  $x = 0$  or  $y = 0$ .
  - Both  $a$  and  $b$  are even.
  - If  $x > 0$ , then  $x^2 \geq 0$ .  
For  $d$  and  $e$ , let  $A$  be subset of a universal set  $U$ .
  - For every set  $A$ ,  $A \cap \bar{A} = \phi$ .
  - There exists a set  $A$  such that  $\bar{A} \subseteq A$ .
  - For every rational number  $r$ ,  $r^2 + r$  is rational.
  - There exists a rational number  $r$  such that  $r^3 = 3$ .
- 20) State each of the following in symbols and negate each statement (or open sentence).
- There exist integers  $a$  and  $b$  such that  $ab > 0$  and  $a + b < 0$ .
  - For all real numbers  $x$  and  $y$ , if  $x + y > 0$ , then  $xy \geq 0$ .
- 21) Determine if each of the following is True or False. Justify you answer.
- $\forall x \in \mathbb{R}, x^2 > 0$
  - $\exists n \in \mathbb{N}, n < 1$
  - $\exists q \in \mathbb{Q}, \frac{1}{q^2} = \frac{1}{2}$
  - $\forall m, n \in \mathbb{Z}, nm > 0$
  - $\forall m, n \in \mathbb{N}, nm > 0$

Not collected Book problems: 1.1, 1.5, 1.7, 1.11, 1.19, 1.41, 2.1, 2.3, 2.5, 2.8, 2.11, 2.17, 2.19, 2.23, 2.33, 2.37, 2.39, 2.45, 2.49