

## HOMEWORK 2

- 1) Let  $x \in \mathbb{R}$ . If  $x^x > 0$ , then  $x^2 + 1 > 0$ .
- 2) Let  $y \in \mathbb{R}$ . If  $y^2 < 0$ , then  $y + 1 > 0$ .
- 3) Prove directly:
  - a) Let  $x \in \mathbb{Z}$ . If  $x$  is even, then  $3x - 4$  is even.
  - b) Let  $x \in \mathbb{Z}$ . If  $x$  is odd, then  $3x - 4$  is odd.
  - c) Let  $n \in \mathbb{Z}$ . If  $n^2 - 1 < 0$ , then  $n - 2$  is even.
- 4) Prove by contrapositive:
  - a) Let  $x \in \mathbb{Z}$ . If  $3x + 1$  is even, then  $x$  is odd.
  - b) Let  $x \in \mathbb{Z}$ . If  $3x + 1$  is odd, then  $x$  is even.
  - c) Let  $n \in \mathbb{Z}$ . If  $n^2 - 1 < 0$ , then  $n - 2$  is even.
- 5) Prove that  $x + y$  is odd if and only if  $x$  and  $y$  are of the opposite parity.
- 6) Prove that  $ab$  and  $a + b$  are the opposite parity if and only if  $a$  is odd or  $b$  is odd.
- 7) Prove that if  $n \in \mathbb{Z}$ , then  $n^2 + n$  is even.
- 8) Let  $m \in \mathbb{Z}$ . Prove that  $3|m$  if and only if  $3|m^2$ .
- 9) Let  $x, y \in \mathbb{Z}$ .
  - a) Prove that if  $3 \nmid x$ , then  $x^2 = 3m + 1$  for some  $m \in \mathbb{Z}$ .
  - b) Using part a, Prove that if  $3 \nmid x$  and  $3 \nmid y$ , then  $3|(x^2 - y^2)$ .
- 10) Let  $x, y \in \mathbb{Z}$ . Prove that  $3|xy$  if and only if  $3|x$  or  $3|y$ .

Not collected Book problems: 3.1, 3.5, 3.7, 3.11, 3.12, 3.20, 3.27, 3.28, 3.29, 3.30, 3.33, 3.39, 4.3, 4.5, 4.7