HOMEWORK 6

For each of numbers 1 through 5 do the following:
a) Write the statement in symbols.
b) Negate the statement and write the negation in words.
c) Prove or Disprove (which is given at the end of each problem).

1) For every non-negative integer n, there exists a non-negative k such that $k < n$. (Prove)

2) There exists a real number a such that for all real numbers b and c, $\frac{a+b}{a+c} = \frac{b}{c}$. (Prove)

3) There exists an integer n such that $3n^2 - 5n + 1$ is even. (Disprove)

4) There exists an integer n such that for all integers m, $-nm < 0$. (Disprove)

5) Every nonzero rational number is the product of two irrational numbers. (Prove)

For problems 6 through 14 Prove or Disprove: (You must clearly state which you are doing and how!)

6) For every two rational numbers a and b with $a < b$, there exists a rational number c such that $a < c < b$.

7) If $x, y \in \mathbb{R}$ and $x^3 < y^3$, then $x < y$.

8) If $x, y \in \mathbb{R}$ and $x^2 < y^2$, then $x < y$.

9) For every two sets A and B, $(A \cup B) - B = A$.

10) Let A be a set. If, for all sets B, $A - B = \phi$, then $A = \phi$.

11) Let A be a set. If, for all sets B, $A \cap B = \phi$, then $A = \phi$.

12) Let $f(x) = x^5 + x^3 + 1$ be a polynomial defined on $\mathbb{R}$. Then $f(x)$ has a zero in $[-1, 1]$.

13) Let $f(x) = x^2 + 4x^6 + 1$ be a polynomial defined on $\mathbb{R}$. Then $f(x)$ has a zero in $[-1, 1]$.

14) For every positive integer n, $n^2 - n + 11$ is prime.

15) Chapter 7 number 71 (follow the instructions given).

Challenge question - do on a separate piece of paper - Chapter 7 number 74.

Not collected Book problems: 7.7, 7.11, 7.13, 7.25, 7.26, 7.27, 7.32, 7.37, 7.41, 7.52, 7.54, 7.60