

HOMEWORK 7

- 1) State if the following relations on $A = \{a, b, c\}$ are reflexive, symmetric, and/or transitive (or not).
 - a) $R_1 = \{(a, a), (b, b), (c, c), (a, b), (b, c)\}$.
 - b) $R_2 = \{(c, c), (a, b), (b, c), (a, c)\}$.
 - c) $R_3 = \{(a, a), (b, c), (c, b), (a, c), (c, a)\}$.

- 2) Create non-empty relations S and T on the set $A = \{r, s, t\}$ that satisfy the following:
 - a) S is Not reflexive, symmetric and transitive.
 - b) T is Reflexive, symmetric and transitive.

- 3) Hwk 8.16.

- 4) Let A be the set of all functions on \mathbb{R} . Define the relation R on A defined by fRg if $f(x) = g(x)$ for all $x \in [0, 1]$. Prove R is an equivalence relation.

- 5) State whether or not each of the following is a function on \mathbb{R} .
 - a) $y = \sqrt{x + \pi}$
 - b) $y = \sqrt{x^4 - 1}$
 - c) $y = x^3 - 3$

- 6) State the (largest possible) domain and range of each of the following functions. (as subsets of \mathbb{R} .)
 - a) $f(x) = \frac{x^2-1}{x-2}$
 - b) $f(x) = \sqrt{x^2 - 9}$
 - c) $f(x) = e^x$

- 7) Are the following functions injective and/or surjective (state yes or no for both)? (The domain and co-domain are given.)
 - a) $f : \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(n) = 2n + 3$.
 - b) $f : \mathbb{Z} \rightarrow \mathbb{Z}$ defined by $f(n) = n + 67$.
 - c) $f : \mathbb{Z} \rightarrow \mathbb{Z}$ defined by $f(n) = 2n + 3$.

- 8) Prove the following functions are bijective.
 - a) $f : \mathbb{R} - \{0\} \rightarrow \mathbb{R} - \{3\}$ defined by $f(x) = \frac{3x+1}{2x}$.
 - b) $f : \mathbb{R} - \{2\} \rightarrow \mathbb{R} - \{4\}$ defined by $f(x) = \frac{4x-1}{x-2}$.