

HOMEWORK

- 1) Prove the following functions are bijective (or state you already did this)
 - a) $f(x) : \mathbb{R} - \{0\} \rightarrow \mathbb{R} - \{\frac{3}{2}\}$ defined by $f(x) = \frac{3x+1}{2x}$.
 - b) $f(x) : \mathbb{R} - \{2\} \rightarrow \mathbb{R} - \{4\}$ defined by $f(x) = \frac{4x-1}{x-2}$.
- 2) Prove the following functions are bijective
 - a) $f(x) : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq -2}$ defined by $f(x) = x^2 - 2$.
 - b) $f(x) : \mathbb{Q} \rightarrow \mathbb{Q}$ defined by $f(x) = 7x - 3$.
- 3) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 + 3x + 1$. You may not use Calculus in the proof, but you can use it help you figure out how to solve it. You should use precalculus algebra.
 - a) Show f is not injective.
 - b) Find all pairs $r_1 \neq r_2$ such that $f(r_1) = f(r_2)$.
 - c) Show f is not surjective.
 - d) Find the range of f .

For the rest of the problems: Let A, B, C be non-empty sets and $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions. Remember, a picture is not a proof, but it can be either an example or counterexample.

- 4)
 - a) Prove: If $g \circ f$ is injective, then f is injective.
 - b) Disprove: If $g \circ f$ is injective, then g is injective.
 - c) Prove or disprove: If $g \circ f$ is surjective, then f is surjective.
 - d) Prove or disprove: If $g \circ f$ is surjective, then g is surjective.
- 5) Prove or disprove:
 - a) If f is injective, then $g \circ f$ is injective.
 - b) If f is surjective, then $g \circ f$ is surjective.
 - c) If g is injective, then $g \circ f$ is injective.
 - d) If g is surjective, then $g \circ f$ is surjective.
- 6) Prove or disprove:
 - a) There exist functions f and g such that f is not injective, but $g \circ f$ is injective.
 - b) There exist functions f and g such that g is not injective, but $g \circ f$ is injective.
 - c) There exist functions f and g such that f is not surjective, but $g \circ f$ is surjective.
 - d) There exist functions f and g such that g is not surjective, but $g \circ f$ is surjective.
- 7) Hwk 9.37.
- 8) . ****More difficult than the others**** Let $g \circ f$ be bijective. Prove:
 - a) If f is onto, then g is one-to-one.
 - b) If g is one-to-one, then f is onto.
 - c) Combining our assumptions from above 'if $g \circ f$ is bijective, f is onto, and g is one-to-one, then what is our conclusion about f and g ?

Not collected Book problems: 9.15, 9.17, 9.19, 9.21, 9.24, 9.29, 9.30, 9.40, 9.43, 9.48($a - f$)