**EXAM 3:** **NAME:**__________________

1)(12pts) State the following:

a) The Monotone Convergence Theorem

b) The definition of a series diverging to $\infty$.

b) The definition of a Cauchy sequence.

2)(14pts) Suppose that $\langle a \rangle$ and $\langle b \rangle$ converge. Prove if $\lim a_n < \lim b_n$, then there exists an $N \in \mathbb{N}$ such that if $n \geq N$ then $a_n \leq b_n$.

3)(14pts) Using the definition of limit, prove

$$\lim_{n\to\infty} \frac{3n+1}{2n-1} = \frac{3}{2}.$$

4)(14pts) Let $x_n = \dfrac{1+n}{1+2n}$. Prove that $\lim\limits_{n\to\infty} x_n$ exists by using Monotone Convergence.

5)(12pts) Prove that if $\langle a \rangle$ converges, then every subsequence of $\langle a \rangle$ converges, and has the same limit as $\langle a \rangle$.

6)(10pts) Suppose that $b \leq L + \varepsilon$ for all $\varepsilon > 0$. Prove that $b \leq L$.

7)(12pts) Suppose that $\sum\limits_{k=1}^{\infty} a_k$ and $\sum\limits_{k=1}^{\infty} b_n$ converge to $A$ and $B$, respectively. Prove

$$\sum_{k=1}^{\infty} (a_k + b_k) = A + B.$$

8)(12pts) Are the following true or false? Give a counterexample or statement of justification.

a) If $\lim a_n = 0$, then $\sum a_n$ converges.

b) If $< x >$ is not monotone, then $< x >$ does not converge.

c) Every bounded sequence converges.