HOMEWORK 13

1) True or false (some of these relate to 2.1 and 2.2 as well as 2.3 and 2.4)

   a) If $\{a_n\} \to a$ and $\{b_n\} \to b$ and $a < b$ then there exists $N \in \mathbb{N}$ such that $n \geq N$ implies $a_n < b_n$.

   b) If $\{a_n\} \to a$ and $\{b_n\} \to b$ and $a \leq b$ then there exists $N \in \mathbb{N}$ such that $n \geq N$ implies $a_n \leq b_n$.

   c) If S is a bounded set of real numbers that contains its Supremum and Infimum, then S is a closed interval.

   d) If $\{x_n\}$ is not monotone then it has no limit.

   e) If $b \leq L + \epsilon$ for all $\epsilon > 0$, then $b \leq l$.

2) Let $x_n = \frac{1}{2n+1} + \frac{1}{2n+2} + \cdots + \frac{1}{3n}$ for all $n \in \mathbb{N}$. Prove the limit of $\{x_n\}$ exists using the monotone convergence theorem.

3) Let $y_n = \frac{3n-1}{8n-2}$ for all $n \in \mathbb{N}$. Prove that y_n converges using the monotone convergence theorem.

4) Let $a_1 = 1$ and $a_n = \sqrt{3a_{n-1} + 4}$ for $n \geq 2$. Prove $a_n < 4$ for all $n \in \mathbb{N}$.

5) Use monotone convergence to prove the following converge and then find the limit using that a subsequence must converge to the same limit as the sequence.

   a) $s_1 = 1$ and $s_n = \frac{1}{4}(s_{n-1} + 5)$ for $n \geq 2$

   b) $s_1 = 2$ and $s_n = \frac{1}{4}(s_{n-1} + 5)$ for $n \geq 2$

6) Suppose $x_1 > -1$ and $x_n = \sqrt{1 + x_{n-1}}$ for $n \geq 2$. Prove that $\{x_n\}$ converges and find its limit.