HOMEWORK 6

(1) Regular Induction:

1) Prove by induction that $2 + 4 + 6 + \cdots + 2n = n(n + 1)$ for all natural numbers n.

2) Find a formula for the sum $3 + 7 + 11 + \cdots + (4n - 1)$. Verify the formula is correct by induction.

3) Let $b, t \in \mathbb{R}$ with $t \neq 1$. Prove $b + bt + bt^2 + \cdots + bt^{k-1} = \frac{b(t^k-1)}{t-1}$ for all positive integers k.

4) Let $a_1 = 0$ and $a_n = 2a_{n-1} + n - 1$ for $n \geq 2$. Prove by induction that $a_n = 2^n - n - 1$ for all $n \in \mathbb{N}$.

(2) General Induction:

5) Prove by induction that $6|(m^3 - m)$ for $m \geq -3$.

6) Prove by induction for $n \geq 5$, $(n + 1)! > 2^{n+3}$.

(3) Strong Induction:

7) In Hwk6.35 (Chartrand) the Fibonacci Numbers are defined. Using that definition prove the following by strong induction:
   a) $F_1 + F_2 + \cdots + F_n = F_{n+2} - 1$

   b) $(F_1)^2 + (F_2)^2 + \cdots + (F_n)^2 = F_n F_{n+1}$.

8) Define the following recursive sequence: $a_1 = 1, a_2 = 5$, and $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 3$. Prove by strong induction that $a_n = 3^n - 2^n$ for all natural numbers n.

9) Define the following recursive sequence: $a_1 = 2, a_2 = 4$, and $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 3$. Prove by strong induction that $a_n = 2^n$ for all natural numbers n.

10) Prove by strong induction, there exists $x, y \in \mathbb{Z}$ such that $2x + 5y = n$ for all $n \geq 4$.