HOMEWORK 8

- 7.2:

0) What do each of the following mean? Which are true and which are false?
   a) $\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, x < y$
   b) $\exists y \in \mathbb{N}, \forall x \in \mathbb{N}, x < y$
   c) $\exists x \in \mathbb{N}, \forall y \in \mathbb{N}, x < y$
   d) $\forall y \in \mathbb{N}, \exists x \in \mathbb{N}, x < y$
   e) $\exists x \in \mathbb{N}, \exists y \in \mathbb{N}, x < y$
   f) $\forall x \in \mathbb{N}, \forall y \in \mathbb{N}, x < y$

1) Explain the difference between homework problems 7.6 and 7.7 from the text. (Other than one is true and the other is false!)

2) State each of the following in symbols:
   a) For every natural number x, there exists a natural number y such that $x < y < 2x$.
   b) There exist two integers a and b such that for all natural numbers n, $a < \frac{1}{n} < b$.
   c) For all real numbers x_0, there exists an $\epsilon \in \mathbb{R}^+$ such that for all $\delta \in \mathbb{R}^+$, $|x - x_0| < \delta \Rightarrow |f(x) - (f(x_0)| < \epsilon$.
   d) Every positive even integer is the sum of two positive even integers.

3) Negate each of the following:
   a) $\exists a \in A, \forall b \in B, a \in C \Leftrightarrow b \in C$
   b) $\forall a, b, c \in \mathbb{R}, \forall y \in \mathbb{R}^+, \exists x \in \mathbb{R}^+, ax^2 + bx + c = 0$
   c) $\forall x_0 \in \mathbb{R}, \forall \epsilon \in \mathbb{R}^+, \exists \delta \in \mathbb{R}^+, |x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \epsilon$
   d) $\exists \delta \in \mathbb{R}^+, \forall \epsilon \in \mathbb{R}^+, \forall x_0 \in \mathbb{R}, |x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \epsilon$

4) Prove $2b$ is true.

5) Disprove $2a$ and $3a$.

- 7.3 Prove or disprove the following:

6) Let A be a set. Then $A \cap B = \phi$ for all sets B if and only if $A = \phi$.

7) Let A, B be sets. If $A \cup B \neq \phi$, then $A \neq \phi$ and $B \neq \phi$.

8) Let A be a set. If $A \neq \phi$, then $A \cup B \neq \phi$ for all sets B.

9) Every even integer can be expressed as the sum of three distinct even integers.

10) If $n \in \mathbb{Z}$, then $3|(n^3 - n)$.

11) There exist two distinct positive integers whose sum exceeds their product.

12) For all positive integers n, $n^2 - n + 41$ is prime.

13) Let $x, y \in \mathbb{R}$. If $x^3 < y^3$, then $x < y$.

14) Let $x, y \in \mathbb{R}$. If $x^4 > y^4$, then $x > y$.

15) Every nonzero rational number is the product of two irrational numbers.

16) There is a real number solution to $x^2 + x = -1$.

17) The equation $x^3 + x + 1$ has a real solution between $x = -1$ and $x = 0$.