HOMEWORK 14

This homework is from 2.1 and 2.2. You may also use concepts from Ch. 1.

1) True or false. Justify your answers.

   a) If $\{a_n\} \to a$ and $\{b_n\} \to b$ and $a > b$ then there exists $N \in \mathbb{N}$ such that $n \geq N$ implies $a_n > b_n$.

   b) If $\{a_n\} \to a$ and $\{b_n\} \to b$ and $a \geq b$ then there exists $N \in \mathbb{N}$ such that $n \geq N$ implies $a_n \geq b_n$.

   c) If S is a bounded set of real numbers that contains its Supremum and Infimum, then S is a closed interval.

   d) If $b \geq L - \epsilon$ for all $\epsilon > 0$, then $b \geq L$.

2) Let S be a nonempty set of real numbers that is bounded above. Prove there exists a sequence $\{a_n\}$ where $a_n \in S$ for all $n \in \mathbb{N}$ and $\{a_n\}$ converges to $\sup S$.

3) Fitzpatrick section 2.1 number 17.

4) Chartrand number 12.41.

5) Using the definition of convergence (or its negation) prove the sequence $\{\frac{2n+1}{n-3}\}$ does NOT converge to 1.

6) Suppose the sequence $\{a_n\}$ converges to a. Prove that this limit is unique.

7) Prove the interval $[-2, 4]$ is closed.

8) Prove $(-\infty, 0]$ is closed.