HOMEWORK 2

1) Let P : 6 is odd and Q : 11 is prime. Determine whether the following are true or false. Justify each part.

a) $P \vee Q$
b) $P \wedge Q$
c) $(\sim P) \wedge Q$
d) $P \Rightarrow Q$
e) $Q \Rightarrow P$
f) $(\sim P) \Rightarrow Q$
g) $P \Leftrightarrow Q$
h) $(\sim P) \vee (\sim Q)$

2) Let $P : \sqrt{10}$ is rational and $Q : \frac{9}{2}$ is rational. Determine whether the following are true or false. Justify each part.

a) $P \vee Q$
b) $P \wedge Q$
c) $P \Rightarrow Q$
d) $Q \Rightarrow P$
e) $(\sim P) \Rightarrow (\sim Q)$
f) $(\sim Q) \Rightarrow (\sim P)$
g) $P \Leftrightarrow Q$
h) $(\sim P) \Leftrightarrow (\sim Q)$

3) In each of the following two open sentences $P(x)$ and $Q(x)$ are defined over a given domain S. Determine all $x \in S$ for which $P(x) \Rightarrow Q(x)$ is true.

a) $P(x) : 3x + 1$ is prime; $Q(x) : 2x + 1$ is prime; $S = \{1, 2, 3, 4\}$
b) $P(x) : x^2 = 9$; $Q(x) : |x| = 3$; $S = \{-3, -2, 0, 1\}$
c) $P(x) : x^2 \geq 4$; $Q(x) : x \geq 2$; $S = \mathbb{R}$
d) $P(x) : x^2 \geq 4$; $Q(x) : x \geq 2$; $S = \mathbb{N}$

4) In each of the following two open sentences $P(x, y)$ and $Q(x, y)$ are defined over a given domain S. Determine all $x \in S$ for which $P(x, y) \Rightarrow Q(x, y)$ is true.

a) $P(x, y) : x^2 - y^2 = 0$; $Q(x, y) : x = y$; $S = \{(1, -1), (3, 2), (4, 4)\}$
b) $P(x, y) : |x| = |y|$; $Q(x, y) : x = y$; $S = \{(-3, 3), (2, 2), (-1, 2)\}$

5) Repeat homework problem three with $P(x) \Leftrightarrow Q(x)$ instead of $P(x) \Rightarrow Q(x)$.

6) Repeat homework problem four with $P(x, y) \Leftrightarrow Q(x, y)$ instead of $P(x, y) \Rightarrow Q(x, y)$.

7) Give truth tables for the following: Label each as a Tautology, Contradiction, or Neither.
   a) $P \wedge (Q \Rightarrow (\sim P))$
   b) $(P \wedge Q) \Leftrightarrow P$ (what is this logically equivalent to?)

8) Verify the pairs of statements are logically equivalent.
   a) $P \Rightarrow (Q \vee R)$ and $(\sim Q) \Rightarrow ((\sim P) \vee R)$
   b) $(P \wedge Q) \Rightarrow R)$ and $(P \wedge (\sim R)) \Rightarrow (\sim Q)$

9) Negate the following: (whenever possible remove the 'not')
   a) Either $x = 0$ or $y = 0$.
   b) Both a and b are even.
   c) If $x > 0$, then $x^2 \geq 0$.
   For d and e, let A be subset of a universal set U.
   d) For every set A, $A \cap \overline{A} = \phi$.
   e) There exists a set A such that $\overline{A} \subseteq A$.
   f) For every rational number r, $r^2 + r$ is rational.
   g) There exists a rational number r such that $r^3 = 3$.

10) State each of the following in symbols and negate each statement (or open sentence).
   a) There exist integers a and b such that $ab > 0$ and $a + b < 0$.
   b) For all real numbers x and y, if $x + y > 0$, then $xy \geq 0$.

11) Determine if each of the following is True or False. Justify you answer.
   a) $\forall x \in \mathbb{R}, x^2 > 0$
   b) $\exists n \in \mathbb{N}, n < 1$
   c) $\exists q \in \mathbb{Q}, \frac{1}{q^2} = \frac{1}{2}$
   d) $\forall m, n \in \mathbb{Z}, nm > 0$
   e) $\forall m, n \in \mathbb{N}, nm > 0$

Not collected Book problems: $2.11, 2.17, 2.19, 2.23, 2.33, 2.37, 2.39, 2.45, 2.49$