

HOMework 2

- 1) Let $x \in \mathbb{R}$. If $x^x > 0$, then $x^2 + 1 > 0$.
- 2) Let $y \in \mathbb{R}$. If $y^2 < 0$, then $y + 1 > 0$.
- 3) Prove directly:
 - a) Let $x \in \mathbb{Z}$. If x is even, then $3x - 5$ is odd.
 - b) Let $x \in \mathbb{Z}$. If x is odd, then $3x - 5$ is even.
 - c) Let $n \in \mathbb{Z}$. If $n^2 - 1 < 0$, then $n - 2$ is even.
- 4) Prove by contrapositive:
 - a) Let $x \in \mathbb{Z}$. If $3x + 1$ is even, then x is odd.
 - b) Let $x \in \mathbb{Z}$. If $3x + 1$ is odd, then x is even.
- 5) Prove that $x + y$ is even if and only if x and y are of the same parity.
- 6) Prove that ab and $a + b$ are the same parity if and only if a and b are both even.
- 7) Prove that if $n \in \mathbb{Z}$, then $n^2 + n + 1$ is odd.
- 8) Prove that for every two distinct real numbers a and b , either $\frac{a+b}{2} > a$ or $\frac{a+b}{2} > b$.
(Note: for two distinct real numbers a and b , either $\frac{a+b}{2} < a$ or $\frac{a+b}{2} < b$ is also true.)

Not collected Book problems: 3.1, 3.5, 3.7, 3.11, 3.12, 3.20, 3.27, 3.28, 3.29, 3.30, 3.33, 3.39