HOMEWORK 5

1) a) Let $f(x) = 2x^6 + x^2 + 1$. Disprove: there exists a $c \in [-1, 1]$ such that $f(c) = 0$.

   b) Disprove: there exists $x, y \in \mathbb{R}^+$ such that $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$.

2) Let $f(x) = x^3 + x + 1$. Prove that there exists a $c \in [-1, 1]$ such that $f(c) = 0$.

3) Prove that if A and B are sets such that $A \cup B \neq \phi$, then $A \neq \phi$ or $B \neq \phi$.

4) Disprove that if A and B are sets such that $A \cap B = \phi$, then $A = \phi$ or $B = \phi$.

5) a) Give sets A, B, and C such that $A \cap B = A \cap C$, but $B \neq C$.
   b) Give sets A, B, and C such that $A \cup B = A \cup C$, but $B \neq C$.
   c) Let A, B, and C be sets. Prove that if $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then $B = C$.

6) Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$. (You are to prove this part of Thm 4.21.)

7) Let A and B be sets. Prove that $A \subseteq B$ if and only if $A \cup B = B$.

8) Let A and B be sets. Prove that if $A - B \neq \phi$, then $A \not\subseteq B$.

Not collected Book problems: $4.27, 4.33, 4.34, 4.37, 4.61, 4.64$
$5.32, 5.33, 5.36, 5.38, 5.41, 5.43, 5.45, 5.46$