HOMEWORK 6

1) Prove by induction that $1 + 3 + 5 + \cdots + (2n-1) = n^2$ for all $n \in \mathbb{N}$.

2) Prove by induction that $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{(n(n+1))^2}{4}$ for all $n \in \mathbb{N}$.

3) Let $s, t \in \mathbb{R}$ and $t \neq 1$. Prove that $s + st + st^2 + \cdots + st^{n-1} = \frac{s(1-t^n)}{1-t}$ for all $n \in \mathbb{N}$.

4) Prove that $n! > 2^n$ for all integers $n \geq 4$.

5) Prove that $3^n > n^2$ for all integers $n \geq 2$.

For each of numbers 6 through 10 do the following:
a) Write the statement in symbols.
b) Negate the statement and write the negation in words.
c) Prove or Disprove (which is given at the end of each problem).

6) There exists an integer n such that for every integer m, $m(n-3) < 1$. (Prove)

7) For all rational numbers a and b with $a < b$, there exists a rational number c such that $a < c < b$. (Prove)

8) There exists an integer a such that for all integers c, $ac \geq 0$. (Prove)

9) If $x, y \in \mathbb{R}$ and $x^2 < y^2$, then $x < y$. (Disprove)

10) For every two sets A and B, $(A \cup B) - B = A$. (Disprove)

Not collected Book problems: 6.2, 6.5, 6.18, 6.19, 7.7, 7.11, 7.13, 7.15