HOMEWORK 7

For problems 1 through 12 **Prove or Disprove** each.

1) There exists an odd integer, the sum of whose digits is odd and the product of whose digits is odd.
2) There exists an even integer, the sum of whose digits is even and the product of whose digits is odd.
3) There exist two distinct positive integers whose sum exceeds their product.
4) Every positive even integer is the sum of two positive even integers.
5) For all $x, y \in \mathbb{R}$, if $x^3 < y^3$ then $x < y$. (You may not use Calculus.)
6) For every positive irrational number b, there exists a rational number a such that $0 < a < b$.
7) Every nonzero rational number is (ie. can be written as) the product of two irrational numbers.
8) There exists a real solution to the equation $3x^6 + x^2 = -1$.
9) There exists a real solution to the equation $x^5 - x^2 = -1$.
10) For every set A, there exists a set B such that $A \cap B = \phi$.
11) For every set A, there exists a set B such that $A \cup B \neq \phi$.
12) Let A be a set. Then, $A \cap B = \phi$ for all sets B if and only if $A = \phi$.

13) For each relation on $A = \{1, 2, 3\}$ state whether it is reflexive, symmetric, and/or transitive.
    a) $R_1 = \{(1,1), (2,1), (2,2), (1,2), (3,3)\}$
    b) $R_2 = \{(1,1), (2,1), (3,1), (1,3), (1,2)\}$
    c) $R_3 = \{(1,1), (2,2), (3,3)\}$
14) Give a relation on $A = \{a, b, c, d\}$ that has the following properties:
    a) R is not reflexive, not symmetric, and not transitive.
    b) S is reflexive, not symmetric, and not transitive.
    c) T is reflexive, symmetric and transitive.
15) Let A be the set of all functions on $\mathbb{R}$. Prove the relation R on A defined by fRg if $f(x) = g(x)$ for all $x \in [0, 1]$, is an equivalence relation.
16) State whether or not each of the following is a function on $\mathbb{R}$.
    a) $y = \sqrt{x^2 + 1}$
    b) $y = \sqrt{x + 1}$
    c) $y^2 = x^2 + 1$
17) State the domain (largest possible) and range for each of the following functions.
    a) $f(x) = \sqrt{x - 1}$
    b) $f(x) = \frac{2x+1}{x-1}$
    c) $f(x) = \ln x$