HOMEWORK 6

1) Let $f : \mathbb{Z} \to \mathbb{Z}$ be a function. Are the following definitions of f injective, surjective, bijective, or none of them? If the function is not injective or not surjective give an example where it is not.
   a) $f(n) = 2n + 1$
   b) $f(n) = n^2$
   c) $f(n) = n + 3$

2) Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Are the following definitions of f injective, surjective, bijective, or none of them? If the function is not injective or not surjective give an example where it is not.
   a) $f(x) = 2x + 1$
   b) $f(x) = x^2$
   c) $f(x) = \frac{1}{x^2+1}$

3) Define sets A and B and functions $f : A \to B$ and $g : B \to A$ such that f is injective but not surjective and g is surjective but not injective.

4) Give four distinct functions that are bijective from the interval $[0, 1]$ to itself.

5) Prove the function $f : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 2}$ defined by $f(x) = x^2 + 2$ is bijective.

6) Prove the function $f : \mathbb{R} - \{-2\} \to \mathbb{R} - \{3\}$ defined by $f(x) = \frac{3x-1}{x+2}$ is bijective.

7) Prove the function $f : \mathbb{Q} \to \mathbb{Q}$ defined by $f(x) = 3x - 1$ is bijective. (You may not 'quote' all linear functions are bijective.)

8) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 + 2x + 2$. (You may use calculus to help you, but not as part of your 'proof'.)
   a) Show that f is not injective.
   b) Find all pairs r_1, r_2 such that $f(r_1) = f(r_2)$. (Hint: r_2 should somehow be a 'function' of r_1.)
   c) Show that f is not surjective.
   d) Find the range of f.

Not collected Book problems: 9.13, 9.15, 9.17, 9.21, 9.43