HOMEWORK 9

For all the problems: Let A, B, C be non-empty sets and $f : A \to B$ and $g : B \to C$ be functions. Remember, a picture is not a proof, but it can be either an example or counterexample.

1) a) Prove: If $g \circ f$ is injective, then f is injective.
   b) Disprove: If $g \circ f$ is injective, then g is injective.
   c) Prove or disprove: If $g \circ f$ is surjective, then f is surjective.
   d) Prove or disprove: If $g \circ f$ is surjective, then g is surjective.

2) Prove or disprove:
   a) If f is injective, then $g \circ f$ is injective.
   b) If f is surjective, then $g \circ f$ is surjective.
   c) If g is injective, then $g \circ f$ is injective.
   d) If g is surjective, then $g \circ f$ is surjective.

3) Prove or disprove:
   a) There exist a functions f and g such that f is not injective, but $g \circ f$ is injective.
   b) There exist a functions f and g such that g is not injective, but $g \circ f$ is injective.
   c) There exist a functions f and g such that f is not surjective, but $g \circ f$ is surjective.
   d) There exist a functions f and g such that g is not surjective, but $g \circ f$ is surjective.

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5) **More difficult than the others** Let $g \circ f$ be bijective. Prove:
   a) If f is onto, then g is one-to-one.
   b) If g is one-to-one, then f is onto.
   c) Combining our assumptions from above 'if $g \circ f$ is bijective, f is onto, and g is one-to-one', then what is our conclusion about f and g?